Uncovered Interest Rate Parity Puzzle: An Explanation based on Recursive Utility and Stochastic Volatility

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August 1, 2008

Abstract

I examine the uncovered interest rate parity puzzle in a two-country economy where agents have recursive preferences. The model rationalizes the anomaly thanks to the presence of two ingredients: preference for the early resolution of risk and stochastic volatility in consumption growth. When U.S. consumption volatility is relatively low, exchange rate variability is closely tied to shocks in U.K. consumption. This is foreign exchange risk for the U.K. investor. At the same time, the preference for the early resolution of risk drives the U.S. interest rate up when U.S. volatility is low, thus solving the puzzle.

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1 Introduction

The uncovered interest rate parity (UIP) puzzle states that high interest rate currencies appreciate over time and therefore pay a positive expected excess return. This empirical finding is consistently confirmed by numerous studies (see, among others, Engel (1996) and Lewis (1995)). I show that the anomaly arises naturally in a two-country model with two ingredients: (i) Epstein-Zin preferences with a preference for the early resolution of risk, and (ii) stochastic volatility in consumption growth.

The economics underlying my results are as follows. Fama (1984) noted that the U.S. minus U.K. (real) interest rate differential can be written as

\[ r_t - r_t^* = p_t + q_t , \]

where \( q_t \) is the expected rate of depreciation on the U.S. (real) exchange rate and, therefore, \( p_t \) is the expected excess return on a “carry trade” which delivers U.K. goods and receives U.S. goods. Backus, Foresi & Telmer (2001) showed that (with conditional lognormality), \( p_t \) and \( q_t \) can be written as

\[ p_t = \frac{Var_t(\log m_{t+1}^*)}{2} - \frac{Var_t(\log m_{t+1})}{2} \]
\[ q_t = E_t \log m_{t+1}^* - E_t \log m_{t+1} , \]

where \( m \) and \( m^* \) are the U.S. and U.K. real pricing kernels, respectively, and, with complete markets, the realized depreciation rate of the U.S. real exchange rate is

\[ d_{t+1} = \log m_{t+1}^* - \log m_{t+1} , \]

so that \( q_t = E_t d_{t+1} \). Fama’s well-known conditions for the resolution of the UIP puzzle are that (i) \( cov(p, q) < 0 \) and (ii) \( var(p) > var(q) \). I show that stochastic volatility and Epstein-Zin preferences are sufficient to have both these conditions satisfied.

Why stochastic volatility? Equation (1) makes it clear that, at least with lognormality, there is no choice. Without variability in the conditional variances, \( p_t \) is a constant and both of Fama’s condition are violated. With stochastic volatility, what is going on is as follows. When U.K. consumption volatility is relatively high then, according to equation (3), variations in the exchange rate will be dominated by variations in U.K. consumption. The U.K. investor views this as ‘exchange rate risk’ and, therefore, requires a positive risk premium in order to hold a security which is long this source of risk. A carry trade that is long U.S Dollars will, therefore, have a positive expected payoff.

Why Epstein-Zin (EZ) preferences? Because the UIP puzzle requires a connection between the conditional variance and the conditional mean of the pricing kernel. With standard time and state separable preferences, the stochastic volatility that is driving the risk premium, \( p_t \),
cannot affect the expected depreciation rate, $q_t$. The result is a violation of Fama’s condition (i). In addition, the separation of risk aversion from intertemporal elasticity of substitution is instrumental in getting the interest rate differential to move in the right direction. Intuitively, when the conditional variance of the U.K. pricing kernel is relatively high — so that the risk premium is positive — the U.K. interest rate will be relatively low if agents have preference for the early resolution of risk.

Figure 1 clarifies the mechanism described above. The trees in panels A and B show how different attitudes toward the timing in the resolution of uncertainty affect utility over states of nature and time. With early resolution of risk, an increase in conditional variance is analogous to moving the agent from the tree bifurcating early in panel A, which has a non-constant conditional mean and zero conditional variance, to the tree bifurcating late in panel B, which has a constant conditional mean and a positive conditional variance. This shock ultimately leaves the agent worse off and, in order to make her indifferent, we could increase the level of the conditional mean, which is equivalent to a decrease in the interest rate.\footnote{In contrast to the standard case of state and time separable utility, with recursive preferences, an increase in the conditional variance of consumption does not necessarily imply a decrease in the level of the interest rate. The usual precautionary savings effect is modified to take into account the role played by the timing of the resolution of uncertainty. The interpretation of Figure 1 is due to Stanley Zin.}

To summarize, the story is this. If U.K. consumption volatility increases relative to that of the U.S., then the U.K. agent views foreign ‘currency’ investments as being riskier than does her U.S. counterpart, because U.K. consumption shocks become more strongly related to exchange rate shocks. The U.K. agent will require a risk premium. This risk premium must be manifest in either a relatively low U.K. interest rate, or an expected appreciation in the U.S. exchange rate, or a bit of both. The facts say that it must be a bit of both. EZ preferences deliver ‘a bit of both’ by (i) allowing stochastic volatility to affect both the first and second moments of the (log) pricing kernels, and (ii) allowing preference for the early resolution of risk to drive down the U.K. interest rate without affecting one-to-one the exchange rate.

Two related studies of the UIP puzzle which pre-date this paper are Bansal and Shaliastovich (2006) (2008) and Verdelhan (2007). Bansal and Shaliastovich analyze the anomaly with recursive preferences but emphasize the importance of long-run risk. In contrast, this paper indicates that long-run risk is not necessary for the UIP puzzle. It argues that stochastic volatility — in tandem with EZ preferences — is sufficient and, in light of equation (3), more directly related to the requisite risk premium. The intuition behind Verdelhan’s model is similar but the economics are very different. In his paper, results are driven by the relative distance from habit consumption levels, which affects the level of risk aversion of the agents. Here instead, risk aversion is constant and a crucial role is played by stochastic volatility in consumption growth.
I calibrate the model to match U.S. monthly consumption data. The implied average level of the real interest rate is 1.3% and the cross-country correlation in consumption is consistent with what we observe in the data. The implied volatility of the depreciation rate on the U.S. real exchange rate is around 22%.

The rest of this paper is organized as follows. Section 2 introduces the model, Section 3 provides a solution to the UIP puzzle, Section 4 delivers the results and Section 5 offers suggestions for further research and concludes.

2 The Model

In this section I describe the preferences of the agents and introduce the process followed by consumption growth in both countries.

2.1 Epstein-Zin Preferences

There is a representative agent in each country who chooses to maximize the recursive utility function given by Epstein and Zin (1989) (1991). The intertemporal utility functions for the U.S. and U.K. agents, $U_t$ and $U^*_t$ respectively, are the solution to the recursive equations:

$$U_t = [(1 - \beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}$$

and

$$U^*_t = [(1 - \beta^*)c_t^\rho^* + \beta^*\mu^*_t(U^*_{t+1})^\rho^*]^{1/\rho^*},$$

where $\beta$ and $\beta^*$ characterize impatience, $\rho$ and $\rho^*$ measure the preference for intertemporal substitution, and the certainty equivalents of random future utility are specified as

$$\mu_t(U_{t+1}) \equiv E_t[U_{t+1}^{\alpha}]/\alpha$$

and

$$\mu^*_t(U^*_{t+1}) \equiv E_t[U^*_{t+1}^{\alpha^*}]/\alpha^*,$$

where $\alpha$ and $\alpha^*$ measure static relative risk aversion (RRA). Both $\alpha$ and $\rho$ are defined for values not greater than one. The relative magnitude of $\alpha$ and $\rho$ determines whether agents prefer early resolution of risk ($\alpha < \rho$), late resolution of risk ($\alpha > \rho$), or are indifferent to the timing of resolution of risk ($\alpha = \rho$). The U.S. marginal rate of intertemporal substitution is

$$m_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{\rho-1} \left( \frac{U_{t+1}}{\mu_t(U_{t+1})} \right)^{\alpha-\rho}.$$

An equivalent expression can be obtained for the U.K. representative agent. Standard time and state separable utility corresponds to the case in which $\alpha = \rho$. 

2.2 A Consumption growth process with stochastic volatility

I specify the dynamics of observed consumption growth with a heteroskedastic AR(1) process. I need a heteroskedastic process to be able to generate a time-varying risk premium and I choose an AR(1) process to allow for non-zero autocorrelation. The U.S. process evolves statistically according to

\[ x_{t+1} = (1 - \phi_x) \theta_x + \phi_x x_t + \nu_t^{1/2} \epsilon_{t+1}^x, \]

where

\[ \nu_{t+1} = (1 - \phi_v) \theta_v + \phi_v \nu_t + \sigma_v \epsilon_{t+1}^v \]

is the process for the conditional volatility of U.S. consumption growth. Similarly, the dynamics for U.K. consumption satisfy

\[ x_{t+1}^* = (1 - \phi_x^*) \theta_x^* + \phi_x^* x_t^* + \nu_t^{1/2} \epsilon_{t+1}^{x^*}, \]

where

\[ \nu_{t+1}^* = (1 - \phi_v^*) \theta_v^* + \phi_v^* \nu_t^* + \sigma_v^* \epsilon_{t+1}^{v^*} \]

I refer to \( \nu_t \) and \( \nu_t^* \) as stochastic volatilities: they will prove essential in the solution of the puzzle. For any \( t \), innovations to consumption growth and stochastic volatility are serially uncorrelated and distributed according to the following multivariate normal:

\[
\begin{bmatrix}
\epsilon_t^x \\
\epsilon_t^v \\
\epsilon_t^{x^*} \\
\epsilon_t^{v^*}
\end{bmatrix}
\sim N
\begin{pmatrix}
0 \\
0 \\
\chi_x \\
0
\end{pmatrix};
\begin{pmatrix}
1 & 0 & \chi_x & 0 \\\n0 & 1 & 0 & \chi_v \\
\chi_x & 0 & 1 & 0 \\
0 & \chi_v & 0 & 1
\end{pmatrix}
\]

I allow for non-zero correlation between respective innovations across countries, and define \( \chi_x \equiv corr(\epsilon_t^x, \epsilon_t^{x^*}) \) and \( \chi_v \equiv corr(\epsilon_t^v, \epsilon_t^{v^*}) \). The process for consumption growth requires that the volatility process be positive, which places further restrictions on the parameters.

Regardless of the specific structure of the economy, with complete financial markets the following first order conditions must hold:

\[ E_t(m_{t+1} R_{t+1}) = 1, \tag{4} \]

and

\[ E_t(m_{t+1}^* R_{t+1}^*) = 1, \tag{5} \]

where \( R_{t+1} \) and \( R_{t+1}^* \) are the gross domestic and foreign one period returns. The nature of this exercise is to make parametric assumptions about the processes followed by the observed domestic and foreign consumption growth, and give sufficient conditions on preference parameters to solve the UIP puzzle. A fully specified general equilibrium model is not needed for
the analysis. In other words, I take observed consumption data as the competitive allocation resulting from the underlying structure of the economy.

It is important to notice that no long-run risk factor enters the process followed by consumption growth. The concept of long-run risk was first introduced by Bansal and Yaron (2004). In their model, both fluctuations in the long-run growth prospects of the economy (the long-run risk) and the time-varying level of economic uncertainty (stochastic volatility) drive financial markets. Bansal and Shaliastovich (2006) extend the long-run risk model to two countries and are able to explain the UIP puzzle emphasizing the importance of the contemporaneous presence of three ingredients: long-run risk, stochastic volatility and early resolution of risk. I argue that long-run risk in consumption growth is not needed to rationalize the puzzle, as it adds a degree of freedom to the analysis but does not capture any essential component of the anomaly.

\section{The Pricing Kernel}

For brevity, the following derivations are provided for the U.S. agent only. Their extensions to the U.K. agent are straightforward. The log of the equilibrium domestic marginal rate of substitution is given by

$$\log(m_{t+1}) = \log(\beta) + (\rho - 1)x_{t+1} + (\alpha - 1)x_{t+1} + (\alpha - 1)\log(W_{t+1} - \log(W_{t+1})),$$

where $x_{t+1} \equiv \log(c_{t+1}/c_t)$ is the log of the ratio of domestic observed consumption in $t + 1$ relative to $t$ and $W_t$ is the value function. The first two terms are standard expected utility terms: the pure time preference parameter $\beta$ and a consumption growth term times the inverse of the negative of the intertemporal elasticity of substitution (IES). The third term in the pricing kernel is a new term coming from EZ preferences.

I work on a linearized version of the real pricing kernel, following the findings of Hansen, Heaton & Li (2005). In particular, I focus on the the value function of each representative agent, scaled by the observed equilibrium consumption level

$$W_t/c_t = \left[(1 - \beta) + \beta(\mu_t(W_{t+1}/c_t))^{\rho} \right]^{1/\rho},$$

where I use the linear homogeneity of $\mu_t$. In logs,

$$w_t = \rho^{-1} \log([1 - \beta + \beta \exp(\rho w_t)],$$

where $w_t = \log(W_t/c_t)$ and $u_t \equiv \log(\mu_t(\exp(w_{t+1} + x_{t+1})))$. Taking a linear approximation of
the right-hand side as a function of $u_t$ around the point $\bar{m}$, I get

$$w_t \approx \rho^{-1} \log[(1 - \beta) + \beta \exp(\rho \bar{m})] + \left[ \frac{\beta \exp(\rho \bar{m})}{1 - \beta + \beta \exp(\rho \bar{m})} \right] (u_t - \bar{m})$$

≡ $\bar{k} + \kappa u_t$

where $\kappa < 1$. Approximating around $\bar{m} = 0$, results in $\bar{k} = 0$ and $\kappa = \beta$, and for the general case of $\rho = 0$, the “log aggregator”, the linear approximation is exact with $\bar{k} = 1 - \beta$ and $\kappa = \beta$.

Similarly to Gallmeyer et al. (2007), the expression for the linearized real pricing kernel is:

$$-\log(m_{t+1}) = -\log(1 - \rho) x_{t+1}$$

$$- \left( \alpha - \rho \right) (\omega_x + 1) v_{t+1}^{1/2} \epsilon_{t+1}^x + \omega_v \sigma_v \epsilon_{t+1}^v - \frac{\alpha}{2} (\omega_x + 1)^2 v_t - \frac{\alpha}{2} \omega_v^2 \sigma_v^2$$

$$= \delta + \gamma_x x_t + \gamma_v v_t + \lambda_x v_{t+1}^{1/2} \epsilon_{t+1}^x + \lambda_v \sigma_v \epsilon_{t+1}^v,$$

where

$$\delta = -\log(1 - \rho) x_{t+1}$$

$$\gamma_x = (1 - \rho) \phi_x ; \ \gamma_v = \frac{\alpha}{2} (\alpha - \rho) (\omega_x + 1)^2$$

$$\lambda_x = (1 - \alpha) - (\alpha - \rho) \omega_x ; \ \lambda_v = - \left( \frac{\alpha}{2} \right) \left( \frac{\kappa (\alpha - \rho)}{1 - \kappa \phi_x} \right) \left( \frac{1}{1 - \kappa \phi_x} \right)^2$$

$$\omega_x = \left( \frac{\kappa}{1 - \kappa \phi_x} \right) \phi_x \ ; \ \omega_v = \left( \frac{\kappa}{1 - \kappa \phi_v} \right) \left[ \frac{\alpha}{2} \left( \frac{1}{1 - \kappa \phi_v} \right)^2 \right].$$

Details for the derivation are provided in Appendix A.

The first two conditional moments of the real pricing kernel are

$$E_t \log m_{t+1} = -\delta - \gamma_x x_t - \gamma_v v_t$$

(9)

and

$$\text{Var}_t \log m_{t+1} = \lambda_v^2 \sigma_v^2 + \lambda_x^2 v_t.$$ 

(10)

The conditional mean depends both on consumption growth and stochastic volatility, whereas the conditional variance is a linear function of current stochastic volatility only. Note that with standard expected utility ($\alpha = \rho$), the pricing kernel collapses to

$$-\log m_{t+1} = -\log(1 - \rho) x_{t+1},$$
and its conditional moments become

\[
E_t \log m_{t+1} = -\hat{\delta} - (1 - \rho)\phi_x x_t
\]

and

\[
\text{Var}_t \log m_{t+1} = (1 - \rho)^2 v_t,
\]

where \(\hat{\delta} = -\log \beta + (1 - \rho)(1 - \phi_x)\theta_x\). When \(\alpha = \rho\), stochastic volatility is not priced as a separate risk source. Indeed, in this case, both the factor loading and the price of risk of stochastic volatility, \(\gamma_v\) and \(\lambda_v\), collapse to zero. EZ preferences allow agents to receive a compensation for taking volatility risk, to which they would not be entitled with standard time-additive expected utility preferences.

3 A solution to the UIP puzzle

I now derive the risk-free interest rate, the expected depreciation rate and the foreign exchange risk premium, and provide an economic interpretation of the mechanism behind the model.

3.1 Risk-free Interest Rate

From equation (4) and the log pricing kernel in equation (7), the continuously compounded one-month risk-free interest rate is

\[
r_t \equiv -\log E_t(m_{t+1}) = r_0 + \gamma_x x_t + r_v v_t
\]

where \(r_0 = \delta - \frac{1}{2} \lambda_x^2 \sigma_v^2\) and

\[
r_v = -\frac{1}{2} \lambda_x^2 + \gamma_v.
\]

The coefficient \(r_v\) governs the covariance between the risk-free rate and stochastic volatility and, without EZ preferences, collapses to the standard precautionary savings coefficient. The calibration section shows that, with levels of autocorrelation in consumption growth consistent with the data, a preference for the early resolution of risk results in \(r_v\) being negative: a positive shock to volatility drives the interest rate down.

3.2 Expected Depreciation, Forward Premium and Risk Premium

I now impose complete symmetry in the coefficients, but allow for imperfect correlation across countries. From equation (3) and Fama’s decomposition, the expected depreciation \(q_t\) is equal
to:

\[ q_t = \gamma_x (x_t - x_t^*) + \frac{\alpha}{2} (\alpha - \rho) \left( \frac{1}{1 - \kappa \phi_x} \right)^2 (v_t - v_t^*). \]  

The forward premium is

\[ f_t - s_t = r_t - r_t^* \]
\[ = \gamma_x (x_t - x_t^*) + r_v (v_t - v_t^*), \]  

where \( f_t = \log F_t \) denote the logarithm of the one-month forward exchange rate and the first equality follows from covered interest parity. Stochastic volatility creates a link between the expected depreciation and the forward premium. Equations (13) and (14) show that with \( \alpha < 0 \), when the agents have preference for the early resolution of risk, a (relatively) low U.S. volatility is associated with (i) an expected appreciation of the U.S. dollar and (ii) a relatively high U.S. interest rate. This is exactly what the puzzle says: high interest rate currencies tend to appreciate.

The risk premium is defined as the expected excess return on a “carry trade” which delivers U.K. goods and receives U.S. goods. Using the processes followed by U.S. and U.K. consumption growths, we have

\[ p_t = f_t - s_t - q_t \]
\[ = -\frac{1}{2} \lambda_x (v_t - v_t^*). \]  

Unlike the expected depreciation rate and the forward premium, the risk premium does not depend on current consumption growth, but only on stochastic volatility: relatively high U.K. stochastic volatility drives the risk premium up.

Recall that, at least with lognormality, stochastic volatility is not an option. It is a requirement. On the contrary, I argue that long-run risk is an option. To see this, note that Bansal and Shaliastovich (2006) risk premium is as follows (with my notation):

\[ p_t^{BS} = -\frac{1}{2} (\lambda_x^2 + \phi_{LR}^2 \lambda_{LR}^2) (v_t - v_t^*), \]  

where \( \phi_{LR} \) is the long-run risk volatility and \( \lambda_{LR} \) is the long-run price of risk. From equation (16) it is clear that, although long-run parameters enter the coefficient of the risk premium, thus affecting its level and variability, its time series depends exclusively on stochastic volatility. If we shut down the long-run risk channel, we can still explain the anomaly; if we shut down the stochastic volatility channel we cannot. The next section further builds on the differences between the models.

### 3.3 The UIP Slope Coefficient

Simple regressions of the currency depreciation rate on the interest rate differential strongly reject UIP. If UIP is satisfied, the slope coefficient of the interest rate differential is equal to
one and the intercept is equal to zero. On the contrary, results typically show evidence of a slope coefficient well below unity, and often negative. In my model, the UIP slope coefficient is equal to

\[ b = \frac{\text{cov}(p + q, q)}{\text{var}(p + q)} = \frac{\gamma_x^2 \text{var}(x_t - x_t^*) + \gamma_v r_v \text{var}(v_t - v_t^*)}{\gamma_x^2 \text{var}(x_t - x_t^*) + r_v^2 \text{var}(v_t - v_t^*)}. \]  

(17)

Stochastic volatility and EZ preferences allow me to obtain a negative UIP slope coefficient under quite general scenarios without the need for long-run risk. Indeed, the covariance between the risk premium and the expected depreciation is

\[ \text{cov}(p_t, q_t) = -\frac{1}{4} \lambda_x^2 \alpha (\alpha - \rho) \left( \frac{1}{1 - \kappa \phi_x} \right)^2 \text{var}(v_t - v_t^*). \]

When \( \alpha < 0 \), it is sufficient to have a coefficient of risk aversion larger than the inverse of the elasticity of intertemporal substitution \( (\alpha < \rho) \) to generate a negative covariance between \( p_t \) and \( q_t \), thus satisfying Fama’s condition (i). Agents prefer the early resolution of risk.

The variances of the expected depreciation and of the risk premium are

\[ \text{var}(q_t) = \gamma_x^2 \text{var}(x_t - x_t^*) + \frac{1}{4} \left( \alpha (\alpha - \rho) \left( \frac{1}{1 - \kappa \phi_x} \right)^2 \right) \text{var}(v_t - v_t^*) \]

and

\[ \text{var}(p_t) = \frac{1}{4} \lambda_x^4 \text{var}(v_t - v_t^*). \]

(18)

(19)

In the next section, I calibrate the model to U.S. data and show that, and show that Fama’s condition (ii), \( \text{var}(p_t) > \text{var}(q_t) \), is satisfied when agents have preference for the early resolution of risk.

3.4 Economic interpretation

It is the interaction between the timing of the resolution of uncertainty and the correlation in consumption growth, both within and across countries, that allows me to resolve the puzzle. To simplify the analysis and to better understand the intuition underlying the model, this section studies the case of zero autocorrelation in consumption growth \( (\phi_x = 0) \) and zero cross-country correlation in stochastic volatility \( (\chi_v = 0) \). This simplification allows me to isolate the effect of the timing of the resolution of uncertainty.

With complete markets, the depreciation rate of the U.S. Dollar is equal to the ratio of the U.S. to the U.K. pricing kernel (see equation (3)). The conditional variability of the
depreciation rate is therefore

\[ var_t(d_{t+1}) = var_t(\log m^*_{t+1}) + var_t(\log m_{t+1}) - 2 \text{cov}_t(\log m^*_{t+1}, \log m_{t+1}) \]

\[ = 2\lambda^2 \sigma^2_v + (1 - \alpha)^2 (v_t + v^*_t) - 2 \chi^1_v v_t^{1/2} v^*_t^{1/2}. \]

(20)

When the conditional volatility of the U.K. pricing kernel is high relative to the one of the U.S., exchange rate variability is closely tied to shocks in U.K. consumption volatility. This represents exchange risk for the U.K. investor who therefore requires a positive premium to hold a security which is long this source of risk. This is clear from the expression of the risk premium, which simplifies to

\[ p_t = -\frac{1}{2} (1 - \alpha)^2 (v_t - v^*_t). \]

Times of relatively high U.K. volatility are associated with a positive expected excess return. The level of risk aversion determines its size and variability (but not its sign): the higher the risk aversion, the larger and the more volatile the risk premium.

A positive risk premium is not enough to resolve the anomaly. The premium has to covary negatively with the expected depreciation rate or, equivalently, the interest rate differential — U.S minus U.K — has to increase when entering a long position in U.S. Dollars pays a positive expected excess return. This is where the joint use of EZ preferences and stochastic volatility produces its effects. Equation (14) becomes

\[ r_t - r^*_t = -\frac{1}{2} ((1 - \alpha)^2 - \alpha(\alpha - \rho)) (v_t - v^*_t). \]

Two terms affect the sign of the interest rate differential. The first term depends solely on risk aversion and represent the usual precautionary savings coefficient in the standard case of time and state separable utility. The second term is a non linear interaction between risk aversion and the timing of resolution of uncertainty. In times of relatively high consumption volatility in the U.K., the anomaly can be explained when the second effect outweighs the first. A sufficient condition for this is that the agents show preference for the early resolution of risk: interest rates are low when consumption volatility is high. In the language of Backus, Foresi & Telmer (2001), when agents prefer the early resolution of risk, the differences in conditional variances and conditional means of the log pricing kernels move in opposite direction (see equation (1) and (2)).

In the next section I relax the assumption of zero autocorrelation in consumption growth and calibrate the model to U.S. data. The economic intuition remains the same but the analysis is complicated by the presence of consumption growth in the expression for the interest rate differential. In particular, the size and variability of the risk premium now depends non-linearly on risk aversion, timing of the resolution of uncertainty and correlation in cross-country consumption growth. Results confirm that a strong preference for the early resolution of risk is sufficient to rationalize the UIP puzzle.
4 Results

4.1 Data and Calibration

I calibrate the model to reproduce the mean, variance and autocorrelation of the consumption growth process specified in Bansal and Yaron (2004) and study the anomaly for U.S. monthly data. This is to emphasize that a model without long-run risk in consumption that matches the first two moments of a model with long-run risk can nonetheless explain the UIP puzzle. The results are shown in Table 1 and details of the derivation can be found in Appendix B.

In particular, the monthly average consumption growth is equal to 0.15%, with a monthly unconditional volatility of 0.8%. The first order autocorrelation in consumption growth is set equal to 4.36%. The parameters of the stochastic volatility process are chosen to match the unconditional volatility and the autocorrelation in consumption growth. The persistence coefficient of the stochastic volatility process is set equal to 0.987.

I set the cross-country correlation in consumption growth to 35%. This value is consistent with Brandt, Cochrane & Santa Clara (2006) who report correlation coefficients between +0.24 and +0.42 for consumption growth between the US and other industrialized countries. I set the discount factor $\beta$ at 0.999 and the coefficient $\bar{m}$ in the log linearization of the wealth-consumption ratio at zero: this allows me to obtain clean expressions for the coefficients $\kappa$ and $\bar{\kappa}$. The stochastic volatility processes are assumed to be independent, i.e. $\chi_v = 0$. This captures the intuition that, in the short run, economies with the same intrinsic features can be hit by unrelated shocks. Nonetheless the model can potentially handle non-zero cross-country correlation in volatilities.

The level of relative risk aversion is equal to 2 and the intertemporal elasticity of substitution is equal to 10. The large IES implicit in the model for a relatively low RRA implies that agents in the economy have a strong preference for the early resolution of risk.

4.2 Findings and Comparative Statics

4.2.1 UIP Slope, Risk-free Interest Rate and Depreciation Rate.

Table 2 reports the main result of the paper. Consistently with the data, the UIP slope coefficient is negative (and equal to -0.4). The annualized average level of the one-month real interest rate is 1.3%. These results match the findings of Ang, Bekaert, and Wei (2007). The implied volatility of the depreciation rate is 22.77%.

The model requires a strong preference for the early resolution of risk. For a given level of risk aversion, the IES needed to obtain a negative slope coefficient increases for smaller levels
of cross-country correlation in consumption growth. A large RRA reduces the need for a large IES, but at the same time dramatically increases the volatility of the depreciation rate. Table 3 reports the minimum IES required by the model to return a negative UIP slope coefficient and Figure 2 shows three-dimensional graphs of the UIP slope coefficient as a function of IES (−1 < ρ < 0.95) and cross-country correlation in consumption growth (0.2 < χx < 1).

There is a tension in the model between the UIP slope coefficient and the volatility of the depreciation rate. A larger RRA for a given IES facilitates the resolution of the UIP puzzle as it sharply increases the factor loading of stochastic volatility, γv, while its effect on the coefficient rv is mitigated by the presence of the consumption price of risk, λx (see equations (8) and (12)). This results in a larger — and negative — ratio between the numerator and the denominator of (17), the equation for the UIP slope coefficient. However, at the same time, a larger RRA increases the unconditional volatility of the depreciation rate. To see this, recall from the usual variance decomposition formula that

\[ \text{var}(d_{t+1}) = Evar_t(d_{t+1}) + \text{var}E_t(d_{t+1}). \]  

The first term, which accounts for more of 99% of the variability of the depreciation rate, increases with the level of risk aversion. A clear example of this mechanism can be seen by taking the unconditional expectation of equation (20) in the previous section. Figure 3 shows how the implied volatility of the depreciation rate changes with the level of risk aversion for different values of cross-country correlation in consumption growth.

### 4.2.2 Long-run Risk and Cross-Country Correlation in Consumption Growth

How can a model without long-run risk explain the UIP puzzle? To show this, I first assume that the cross-country correlation in consumption growth is very high and study the implications on the results as a function of within-country autocorrelation. This is an unrealistic case, since data suggest low cross-country correlation in consumption growth, but with this simplification I can derive expressions that are easy to interpret and highlight the differences with the work of Bansal and Shaliastovich (2006). In this case,

\[ \text{std}(q_t) - \text{std}(p_t) \approx -\frac{1}{2} \left( \lambda_x^2 - \alpha(\alpha - \rho) \left( \frac{1}{1 - \kappa \phi_x} \right)^2 \right) \text{std}(v_t - v_t^*) \]
\[ = r_v \text{std}(v_t - v_t^*) \]  

and Fama’s condition (ii) is satisfied whenever \( r_v < 0 \). Figure 4 shows the coefficient \( r_v \) as a function of \( \alpha \) and \( \rho \), assuming a very high level of autocorrelation in the consumption growth. When relative risk aversion is large enough, an increasing number of values for \( r_v \) is positive. In order to avoid this possibility, IES has to be larger than one. This result is similar in spirit to the one obtained by Bansal and Shaliastovich. In their specification, it is the autocorrelation
in the long-run risk factor — and not the autocorrelation in consumption growth — that enters the coefficient $r_v$. By definition, the long-run risk process is highly persistent and this is why their models requires $IES > 1$.

What happens when the autocorrelation in consumption growth is low? Figure 5 shows that, as far as $r_v < 0$ is concerned, I need not take a stand on $IES$ being larger than one. When the coefficient of relative risk aversion is larger than one, any value of $\rho$ will do. In sum, if cross-country correlation in consumption growth was high I would need only check Fama’s condition (i), which is satisfied when agents prefer the early resolution of risk. Again, this is similar to what happens in Bansal and Shaliastovich, with the usual caveat that in my model, cross-country correlation in consumption growth — and not cross-country correlation in long-run risk — affects the expected depreciation rate. Bansal and Shaliastovich follow Colacito and Croce (2005) and assume perfect covariance between the innovations to the long-run risk processes, which implies a very high covariation between the long-run risk processes. This is why they can disregard the contribution to the volatility of the depreciation rate coming from the variance of the long-run risk factors.

In my model, consumption growth — and not long-run risk — enters the expected depreciation rate. I need not make any assumption on the level cross-country correlation: data tell us it is in the order of 30-40%. This imposes a lower bound to the value of $\text{var}(x_t - x_t^*)$ equal to 1.2-1.4 times the variance of the consumption growth process (either one, since I have assumed symmetry in the coefficients). A strong preference for the early resolution of risk lowers the variability of the depreciation rate and increases the variability of the risk premium, thus satisfying Fama’s condition (ii). To see this, notice that equation (18) shows that the variance of the expected depreciation rate depends both on the variability of the consumption growth differential and the variability of the stochastic volatility differential, with a coefficient of proportionality equal to the square of the respective factor loading, $\gamma_x$ and $\gamma_v$. Equation (19), on the other hand, shows that the variance of the risk premium is proportional to the price of risk of consumption growth, $\lambda_x$. A high level of IES helps satisfying Fama’s condition (ii), since it lowers $\gamma_x$ and $\gamma_v$ while increasing $\lambda_x$.

### 4.2.3 Robustness to Different Levels of Autocorrelation in Consumption Growth

There is a large debate in the literature about the process followed by consumption growth. If we only look at quantity data, it is hard to reject the hypothesis that consumption growth is a pure $i.i.d.$ process. Given the difficulties in the specification of the process, it seems interesting to analyze how the UIP slope coefficient $b$ varies with the persistence in consumption growth. In order to do so, I fix IES and cross-country correlation and let $\phi_x$ vary. Figure 6 plots the coefficient $b$ as a function of $\phi_x$, for different levels of risk aversion. For $\phi_x < 0$, the UIP slope coefficient decreases monotonically as $\phi_x$ increases. For $\phi_x > 0$, it first rises from its
minimum level reached at $\phi_x = 0$, and then decreases again for high values of $\phi_x$. The higher the risk aversion, the sooner the slope coefficient restarts falling. Yet this is another difference with Bansal and Shaliastovich. Since the long-run risk factor is modeled as a highly persistent component of consumption growth, they essentially focus on the extreme right of Figure 6, whereas I focus on its middle part, where the autocorrelation is around zero.\footnote{More precisely, they focus on the case in which the autocorrelation in the long-run risk factor is high. Again, in my model, consumption growth plays the role that long-run risk has in Bansal and Shaliastovich (2006).}

The reason why $b < 0$ when $|\phi_x|$ is small can be seen observing equation (17). A small persistence lowers the impact of consumption growth on the UIP slope coefficient, therefore giving more scope to the role played by stochastic volatility. Without the need for a persistent long-run risk component in consumption growth, I can reproduce the anomaly for levels of autocorrelation in consumption growth that are consistent with what we observe in the data. All I need are EZ preferences, which allow me to price stochastic volatility.

5 Conclusion

I study the economic foundations of the UIP puzzle and show that the anomaly naturally arises in a two-country model with two ingredients: stochastic volatility and a strong preference for the early resolution of risk. EZ preferences allow me to price stochastic volatility and create a wedge between the expected depreciation and the interest rate differential. As a consequence, the UIP slope coefficient can be different from one or even negative for suitably chosen preference parameters.

In light of the fact that the currency risk premium is a function of higher order moments of the domestic and foreign pricing kernels (and therefore of their variances only in the case of lognormality), I argue that the puzzle can be solved without long-run risk components in consumption growth. I believe this approach has the advantage to rely less on variables that are intrinsically hard to measure while giving a more intuitive explanation of the puzzle based on the relative level of consumption volatility across countries.

The simple calibration exercise in this paper is not a good substitute for a more rigorous simulation exercise and its quantitative implication should be further investigated. Future research will carefully address this issue and further explore the trade-offs between preference parameters, UIP slope coefficient and implied moments of interest and depreciation rates.

Finally, this paper provides an explanation of the anomaly based on purely real factors — consumption growth and stochastic volatility — and deliberately omits monetary policy. As a consequence, this model is completely mute about outstanding issues on the relative importance of real and nominal factors in the UIP puzzle (see Lustig and Verdelhan (2007),
Burnside (2007) and Burnside et al. (2006)). Current research is carefully investigating this aspect taking into account monetary policy rules and inflation processes together with real factors.
References


Appendix A: Pricing Kernel Linearization

In this Appendix I linearize the pricing kernel \( m \) using the method of undetermined coefficients. Given the state variables and the log-linear structure of the model, conjecture a solution for the value function of the form,

\[
\begin{align*}
    w_t &= \bar{\omega} + \omega_x x_t + \omega_v v_t, \\
    w_{t+1} + x_{t+1} &= \bar{\omega} + (\omega_x + 1)x_{t+1} + \omega_v v_{t+1}
\end{align*}
\]

and, using the properties of normal random variables, \( u_t \) can be expressed as

\[
\begin{align*}
    u_t &= \log(\mu_t(\exp(w_{t+1} + x_{t+1}))) \\
    &= \log(E_t[\exp(w_{t+1} + x_{t+1})^{\alpha}]]^{\frac{1}{\alpha}}) \\
    &= E_t[w_{t+1} + x_{t+1}] + \frac{\alpha}{2} \text{Var}_t[w_{t+1} + x_{t+1}] \\
    &= \bar{\omega} + (\omega_x + 1)(1 - \phi_x)\theta_x + \omega_v(1 - \phi_v)\theta_v + (\omega_x + 1)\omega_x x_t + \omega_v \phi_v v_t \\
    &\quad + \frac{\alpha}{2}(\omega_x + 1)^2 v_t + \frac{\alpha}{2} \omega_v^2 \sigma_v^2.
\end{align*}
\]

Using the above expression, I solve for the value-function parameters by matching coefficients.

\[
\begin{align*}
    \omega_x &= \kappa(\omega_x + 1)\phi_x \\
    \Rightarrow \omega_x &= \left(\frac{\kappa}{1 - \kappa \phi_x}\right) \phi_x \\
    \omega_v &= \kappa(\omega_v \phi_v + \frac{\alpha}{2}(\omega_x + 1)^2) \\
    \Rightarrow \omega_v &= \left(\frac{\kappa}{1 - \kappa \phi_v}\right) \left[\frac{\alpha}{2} \left(\frac{1}{1 - \kappa \phi_x}\right)^2\right] \\
    \bar{\omega} &= \frac{\kappa}{1 - \kappa} + \frac{1}{1 - \kappa} \left[(\omega_x + 1)(1 - \phi_x)\theta_x + \omega_v(1 - \phi_v)\theta_v + \frac{\alpha}{2} \omega_v^2 \sigma_v^2\right].
\end{align*}
\]

The solution allows me to simplify the term \( \log(W_{t+1} - \log \mu_t(W_{t+1})) \) in the pricing kernel in equation (6):

\[
\begin{align*}
    \log W_{t+1} - \log \mu_t(W_{t+1}) &= w_{t+1} + x_{t+1} - \log \mu_t(w_{t+1} + x_{t+1}) \\
    &= (\omega_x + 1)[x_{t+1} - E_t x_{t+1}] + \omega_v[v_{t+1} - E_t v_{t+1}] \\
    &\quad - \frac{\alpha}{2}(\omega_x + 1)^2 \text{Var}_t[x_{t+1}] - \frac{\alpha}{2} \omega_v^2 \text{Var}_t[v_{t+1}] \\
    &= (\omega_x + 1)v_t^{1/2} \varepsilon_{t+1} + \omega_v \varepsilon_{t+1} - \frac{\alpha}{2}(\omega_x + 1)^2 v_t - \frac{\alpha}{2} \omega_v^2 \sigma_v^2.
\end{align*}
\]

Equation (7) follows by collecting terms.
Appendix B: Consumption Growth Process Calibration

The consumption growth process of Bansal and Yaron (2004) is summarized by the following three equations:

\[
\begin{align*}
    l_{t+1} &= \rho l_t + \varphi e \sigma_t e_{t+1}, \\
    x_{t+1} &= \mu + l_t + \sigma_t \eta_{t+1} \\
    \sigma_{t+1}^2 &= \sigma^2 + \nu_1 (\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1},
\end{align*}
\]

where \((\eta_t; e_t; w_t)\) are i.i.d. mean-zero, variance-one, normally distributed innovations. Consumption growth contains a low-frequency component \(l_t\) (the long-run effect) and is heteroskedastic, with conditional variance \(\sigma_t^2\). These two state variables capture time-varying growth rates and time-varying economic uncertainty. The calibrated values are reported in Table (4).

I match the mean of the consumption growth process specified by Bansal and Yaron with the mean of the consumption growth process specified in my model:

\[
E(x_t) = EE_t(x_t) = \mu \equiv \theta_x.
\]

The unconditional variance of the long-run component is

\[
\text{var}(l_t) = \rho_t^2 \text{var}(l_{t-1}) + \varphi^2 \text{var}(\sigma_t e_{t+1}).
\]

The process \(l_t\) is assumed to be stationary, therefore \(\text{var}(l_t) = \text{var}(l_{t-1})\), and

\[
\text{var}(l_t) = \frac{\varphi^2 \sigma^2}{1 - \rho_t^2},
\]

where the last equality follows from the fact that

\[
\text{var}(\sigma_t e_{t+1}) = EE_t(\sigma_t^2 e_{t+1}^2) - (EE_t(\sigma_t e_{t+1}))^2 = \sigma^2.
\]

The unconditional variance of the consumption growth process is

\[
\text{var}(x_t) = \text{var}(l_t) + \text{var}(\sigma_t \eta_{t+1}) = \frac{\varphi^2 \sigma^2}{1 - \rho_t^2} + \sigma^2 = 6.37 \times 10^{-5}.
\]

A similar calculation gives the expression for the autocorrelation in consumption growth, i.e.

\[
\text{Autocorr}(x_{t+1}, x_t) = \frac{\text{cov}(x_{t+1}, x_t)}{\text{var}(x_t)} = \frac{\rho_t \varphi^2 \sigma^2}{\varphi^2 \sigma^2 + \sigma^2(1 - \rho_t^2)} = 4.36\% \equiv \phi_x.
\]

In my model, the unconditional variance of consumption growth is equal to \(\frac{\theta_v}{1 - \phi_v^2}\). Given the value of \(\phi_x\) obtained above, and matching the unconditional variances of the two consumption growth process, the mean \(\theta_v\) of the stochastic volatility process \(v_t\) is obtained as follows

\[
\theta_v = \text{var}(x_t)(1 - \phi_x^2) = 6.3553 \times 10^{-5}.
\]

Finally, the autocorrelation of the stochastic volatility process and its volatility are obtained by matching the relevant coefficients, i.e. \(\phi_v = \nu_1\) and \(\sigma_v = \sigma_w\).
Table 1: Calibration parameters: Consumption growth, stochastic volatility and rate of time-preference

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time preference parameter $\beta$</td>
<td>0.999</td>
</tr>
<tr>
<td>Mean of consumption growth $\theta_x$</td>
<td>0.0015</td>
</tr>
<tr>
<td>Autocorrelation in consumption growth $\phi_x$</td>
<td>0.0436</td>
</tr>
<tr>
<td>Mean of stochastic volatility $\theta_v$</td>
<td>6.3553e-5</td>
</tr>
<tr>
<td>Autocorrelation in stochastic volatility $\phi_v$</td>
<td>0.987</td>
</tr>
<tr>
<td>Volatility of market variance $\sigma_v$</td>
<td>6.5e-6</td>
</tr>
<tr>
<td>Correlation of consumption shocks $\chi_x$</td>
<td>0.35</td>
</tr>
<tr>
<td>Correlation of volatility shocks $\chi_v$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Calibration parameters: Risk aversion, intertemporal elasticity of substitution, UIP slope coefficient, risk-free rate and volatility of the depreciation rate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution</td>
<td>10</td>
</tr>
<tr>
<td>UIP slope coefficient</td>
<td>-0.395</td>
</tr>
<tr>
<td>Average real interest rate (annualized)</td>
<td>1.294%</td>
</tr>
<tr>
<td>Volatility of the depreciation rate (annualized)</td>
<td>22.773%</td>
</tr>
</tbody>
</table>
Table 3: Intertemporal Elasticity of Substitution and the UIP Slope Coefficient. Minimum IES required to obtain a negative coefficient for given levels of risk aversion \((1-\alpha)\) and cross-country correlation in consumption growth \(\chi_x\)

<table>
<thead>
<tr>
<th>(\chi_x)</th>
<th>0%</th>
<th>35%</th>
<th>75%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha = -1)</td>
<td>7.926</td>
<td>6.404</td>
<td>4.005</td>
<td>(\alpha &lt; \rho)</td>
</tr>
<tr>
<td>(\alpha = -3)</td>
<td>2.110</td>
<td>1.681</td>
<td>1.018</td>
<td>(\alpha &lt; \rho)</td>
</tr>
<tr>
<td>(\alpha = -5)</td>
<td>0.969</td>
<td>0.760</td>
<td>0.453</td>
<td>(\alpha &lt; \rho)</td>
</tr>
<tr>
<td>(\alpha = -7)</td>
<td>0.541</td>
<td>0.422</td>
<td>0.257</td>
<td>(\alpha &lt; \rho)</td>
</tr>
</tbody>
</table>

Table 4: Bansal and Yaron Baseline Calibration Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of consumption growth (\mu)</td>
<td>0.0015</td>
</tr>
<tr>
<td>Persistence in long-run risks (\rho_l)</td>
<td>0.979</td>
</tr>
<tr>
<td>Volatility loading of long-run risk (\varphi_e)</td>
<td>0.044</td>
</tr>
<tr>
<td>Mean of stochastic volatility (\sigma)</td>
<td>0.0078</td>
</tr>
<tr>
<td>Autocorrelation in stochastic volatility (\nu_1)</td>
<td>0.987</td>
</tr>
<tr>
<td>Volatility of market variance (\sigma_w)</td>
<td>2.33e-6</td>
</tr>
</tbody>
</table>
Figure 1: The role of the timing of the resolution of uncertainty. Panel A and B show the case of preference for the early and the late resolution of risk, respectively ($\alpha < \rho$ vs. $\alpha > \rho$). The lower trees reproduce in a more extended way the same trees in the upper part of the Figure to emphasize the differences between the two cases. The vertical dotted line represent the moment when utility is evaluated. In panel A, where the uncertainty is resolved early, the conditional mean varies and the conditional variance is zero. In Panel B, where the uncertainty is resolved late, the conditional mean is constant and the conditional variance is positive. Given preference for the early resolution of risk, a positive shock to the conditional variance is equivalent to moving the agent from Panel A to Panel B.
Figure 2: UIP slope coefficient $b$ as a function of $\rho$ and $\chi_x$. Relative risk aversion is set at 2, 4, 6 and 8.
Figure 3: Annualized volatility of the deprecation rate as a function of relative risk aversion for different values of cross-country correlation in consumption growth: 0%, 35%, 75% and 100%.
Figure 4: Coefficient \( r_v \) as a function of \( \alpha \) and \( \rho \). The autocorrelation in consumption growth \( \phi_x \) is set at 0.99. Relative risk aversion is \( 1 - \alpha \) and intertemporal elasticity of substitution is \( 1/(1 - \rho) \).
Figure 5: Coefficient $r_v$ as a function of $\alpha$ and $\rho$. The autocorrelation in consumption growth $\phi_x$ is set at 4.36%. Relative risk aversion is $1 - \alpha$ and intertemporal elasticity of substitution is $1/(1 - \rho)$.
Figure 6: UIP slope coefficient $b$ as a function of $\phi_x$. Risk aversion is set at 2, 4, 6 and 8.