Pay-as-You-Wish Pricing

Yuxin Chen

Oded Koenigsberg

Z. John Zhang

October 30, 2009

1Yuxin Chen is Polk Brothers Professor of Marketing at the Kellogg School of Management, Northwestern University, Evanston, IL 60208 (email: yuxin-chen@kellogg.northwestern.edu), Oded Koenigsberg is Meyer Feldberg Associate Professor at the Graduate School of Business, Columbia University, (email: ok2018@columbia.edu) and Z. John Zhang is Murrel J. Ades Professor of Marketing, The Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania 19104, email: zjzhang@wharton.upenn.edu
Abstract

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Firms frequently use a curious pricing mechanism called “pay as you wish” pricing (PAYW). When PAYW is used, a firm lets consumers decide what a product is worth to them and how much they want to pay to get the product. This practice has been observed in a number of industries. In this paper, we theoretically investigate why and where PAYW can be a profitable pricing strategy relative to the conventional “pay as asked” pricing strategy (PAAP). We show that PAYW has a number of advantages over PAAP such that it is well suited for some industries but not for others. These advantages are: 1) PAYW helps a firm to maximally penetrate a market; 2) it allows a firm to price discriminate among heterogeneous consumers; 3) it helps to moderate price competition. We derive conditions under which PAYW dominates PAAP and discuss ways to improve the profitability of PAYW.

Key Words: Pricing Strategy, Competitive Price Discrimination, Self-determined price
1 Introduction

Pricing a product or service is typically a seller’s responsibility or in some cases a joint responsibility for the seller and the buyer if the seller allows, or if the buyer insists on, haggling. As the seller, it is not always easy to execute that responsibility. At the point of sales, the seller wants to charge the buyer as much as feasible, but the buyer wants to pay as little as possible. There lies the conflict in every business that frequently poisons a seller-buyer relationship. It is the conflict that every seller struggles to deal with and every buyer complains about. Therefore, it is not surprising that when Radiohead, the English alternative rock band, announced on October 9, 2007, that it would let fans to decide how much they would pay, if anything, for downloading its new album *In Rainbows*, it immediately caught the media’s attention as well as the imagination of sellers and buyers alike in the marketplace. With this seemingly novel pay-as-you-wish pricing mechanism (PAYW), the band does not have to sweat over what price it charges for its album and fans have nothing to complain about the price they pay.

Of course, PAYW is not new. For ages, street musicians have used this pricing mechanism to make a living; many churches have left it to worshippers to put whatever offerings they like in a collection basket; museums, such as the Metropolitan Museum of Art in New York city, and other charitable organizations routinely let visitors and donors to decide how much they pay. There are many more examples like these where firms relinquish their role as the price setter to consumers. In this paper, we take a first look at this pricing mechanism to see how and where it may work.

The practice of PAYW raises many questions. First, can PAYW only be used profitably in an industry with zero or low marginal costs? One would be tempted to say yes based on the afore-mentioned examples, except that in the restaurant business, where the marginal cost can be substantial, PAYW is frequently used, too. The upscale restaurant *Just Around the Corner* in London, for instance, has been operating profitably under this pricing mechanism for the past twenty two years. One World Cafè in Salt Lake City, Utah, is also one such thriving restaurant. Then, a deeper question is: how do marginal costs play a role in determining whether PAYW
is a more profitable pricing mechanism than, say, the commonly used “pay as asked” pricing mechanism (PAAP)?

Second, how do consumers play a role in the profitability of PAYW? For instance, is it more likely that PAYW would dominate PAAP as a pricing mechanism if there is a higher concentration of higher willingness to pay customers in the market, or a higher concentration of low willingness to pay customers is actually more conducive to the profitability of PAYW? Finally, is it better to use PAYW, instead of PAAP, in a more competitive industry or in a less competitive industry? Our answers to these questions will help us to articulate the special role of PAYW in the pricing toolkit. They will also help us to shed light on the observed practices of pay-as-you-wish pricing.

In this paper, we arrive at our answers to these questions through analyzing a series of nested models. Our analysis will help us to achieve three research objectives. First, our theoretical modeling allows us to identify the advantages of PAYW over PAAP as a pricing mechanism so as to accord it a special role in the pricing toolkit. Our analysis shows that PAYW can help a firm to achieve maximal market penetration, implement price discrimination, and moderate price competition. Second, by deriving the conditions under which PAYW dominates PAAP, we show that PAYW can be a profitable pricing mechanism in industries where there is a sufficient number of fair-minded customers; where the distribution of consumers is skewed toward the low end in terms of consumer willingness to pay; and where the marketplace is very competitive due to low product differentiation. Our analysis further suggests that zero or very low marginal costs are not necessary for the application of PAYW. Third, our analysis shows that requiring or suggesting a minimum price is a good way to improve the profitability of PAYW, as firms have done in practice.

Our research contributes to a growing literature in marketing and economics that explores the strategic pricing implications of fair-minded customers (e.g. Fehr and Schmidt 1999; Feinberg, Krishna and 2002; Cui, Raju, and Zhang 2007). Our focus here is on how fairness interacts with other factors such as marginal costs, number of free-loaders, product differentiation, etc., to make PAYW as a compelling, profitable pricing mechanism. Our research also contributes to the literature on competitive price discrimination where numerous studies explore how firm-initiated price
discrimination can intensify price competition to the detriment of competing firms (e.e. Thisse and Vives, 1988; Fudenberg and Tirole 2000; Shaffer and Zhang 1995, 2000 and 2002). In contrast, we show that while PAYW allows competing firms to implement price discrimination, it can benefit all competing firms, as consumer-initiated price discrimination, by moderating price competition.

In what follows, we start with a simple model and then gradually add complications in successive four sections to explore how various factors may affect the profitability of PAYW. We conclude in Section 7.

2 A Simple Model

The use of PAYW for “In Rainbows” album was apparently quite successful financially for Radiohead. According to comScore, a global Internet information provider, 40% of the US downloaders paid an average of $8.05 for each download, while 36% of worldwide downloaders paid $6 on average. Radiohead subsequently disputed those numbers, hinting that more fans have paid. What is not in dispute is the fact that many loyal fans paid up even though they did not have to and that the low marginal cost for each download was conducive to using such a pricing scheme. Therefore, our modeling will start with two basic factors driving the success of PAYW: fair-minded customers and a low marginal cost.

Consider a market where a firm (such as Radiohead) sells a product (or album). We assume that each consumer purchases at most one unit of the product and that each consumer derives a different level of consumption utility from the product, which we denote with \( r \). Here, \( r \) is also a consumer’s willingness to pay for the product. To model consumer heterogeneity, we assume that \( r \) is a random variable drawn from a probability density function \( \phi (r) \), defined over the domain \([0, 1]\), with the corresponding cumulative distribution function \( \Phi (r) \). We normalize the total market

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1 For the original report, see http://ir.comscore.com/releasedetail.cfm?ReleaseID=273165.
2 According to industry insiders, “even utilizing those figures Radiohead most likely did considerably better financially than if a major label released the album for them.” See http://www.mp3newswire.net/stories/7002/radiohead-comscore.html.
3 We use the terms product and service interchangeably throughout the paper.
size to one and assume that both the firm and consumers are risk neutral. We further assume that the firm incurs a marginal production cost of $c$ per unit, which is a common knowledge, with $0 \leq c < 1$. To sell the product to consumers, the firm can choose one of two pricing strategies: PAAP or PAYW.

When the firm chooses PAAP, it sets a price denoted by $p$, and a consumer will purchase the product if her consumption utility exceeds that price$^4$. Thus, a consumer with $r$ purchases the product if and only if $r \geq p$. The firm’s profits are simply given by

$$\pi_U = [1 - \Phi(p)](p - c).$$

(1)

When the firm follows PAYW, it does not set a price for the product. Instead, each consumer decides the amount she PAYW. We assume for now that all customers are fair-minded in that they never pay a price that is lower than the firm’s marginal costs $c$. In other words, if they are not willing to pay a price higher than $c$, they will simply not purchase. This assumption would help us to isolate the role of marginal costs in adopting PAYW. Later, we will relax this assumption to allow some consumers to pay less than the marginal cost so that the firm actually incurs a loss in selling to them. Thus, in our basic model, only consumers whose consumption utilities exceed the marginal cost $c$ purchase the product. When they purchase the product, we assume that a consumer with consumption utility $r$ PAYW the amount of $c + \lambda(r - c)$, where we have $0 \leq \lambda \leq 1$. Here, the consumer’s payment is always no larger than $r$ and, depending on $\lambda$, the consumer may pay up to $r$. Thus, $\lambda$ is the fraction of surplus a consumer is willing to pass on to the firm voluntarily as a “fair” or “equitable” compensation to the firm, and hence it captures the sense of fairness or generosity on the part of consumers in the market$^5$. As each consumer’s consumption utility is different, each consumer end up paying a different price. The firm’s profits are given by

$$\pi_p = \int_c^1 \lambda(r - c)\phi(r)dr.$$  

(2)

From the expression above, we see that consumer payments and therefore the firm’s profits

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$^4$We leave channel issues to a future paper to focus on direct comparisons between two pricing regimes.

$^5$The formulation of the equitable division of the profit potential here is the same as in Cui, Raju, and Zhang (2007).
increase with $\lambda$ under PAYW. For example, when $\lambda = 0$, all consumers who buy the product pay the marginal cost $c$ and the firm’s profits are zero. When $\lambda = 1$, every consumer PAYW her individual consumption utility $r$ and the firm generates the greatest possible profits (the first-degree price discrimination). Furthermore, the firm’s profits also depend on how consumers’ utilities are distributed. We now proceed to build on this basic setup and to explore various factors that drive the profitability of PAYW.

### 3 Is a Low Marginal Cost a Precondition for Adopting PAYW?

Intuitively, the answer to this question is “yes,” as $c$ is in the payoff functions regardless of which pricing strategy the firm adopts. However, this answer might be too simplistic, as we do observe that high marginal cost firms, such as restaurants, also use PAYW. To investaget the role of marginal costs explicitly, we start with an analysis of the simple case in which consumers’ utilities are distributed uniformly with $\phi(r) = 1$ and $\Phi(r) = r$. In this case, if the firm chooses PAAP, the firm sets its price $p$ to maximize its profits $\pi_u^1 = (1 - p)(p - c)$. The optimal value of $p$ and the profits are given by

$$p^1_u = \frac{1 + c}{2}, \quad \pi^1_u = \frac{(1 - c)^2}{4}. \quad (3)$$

If the firm adopts the PAYW strategy, we can use equation (2.2) to obtain the firm’s profits as

$$\pi^1_p = \frac{\lambda}{2}(1 - c)^2. \quad (4)$$

As the firm adopts PAYW only when $\pi^1_p > \pi^1_u$, we can show that PAYW is optimal if and only if $\lambda > 0.5$. In other words, when consumers’ consumption utilities are drawn from a uniform distribution, the firm’s choice of its pricing strategy depends only on the fairness parameter $\lambda$ and does not at all depend on the marginal cost. This implies, rather surprisingly, that a low marginal cost is not necessarily a pre-condition for a firm to adopt PAYW. With the right conditions, a firm can benefit from PAYW even if it incurs a high marginal cost of production.

The right condition here is clearly the uniform distribution for $r$. With the uniform distribution,
the demand in the market is a linear function of price, i.e., the marginal change of demand with regard to a unit price change is a constant. When this is not the case, or when the marginal change of demand is either increasing or decreasing with price, the firm’s choice between PAAP and PAYW will depend on both $\lambda$ and $c$. In that case, does a higher marginal cost always reduce a firm’s incentive to adopt PAYW?

To investigate the role of marginal costs further, we can specify a more general distribution function. Consider, for instance, that consumers’ consumption utilities are generated from a trapezoid distribution function with $\phi(r) = a + 2(1-a)r$ and $\Phi(r) = ar + (1-a)r^2$, where $0 \leq a \leq 2$. Note that when $a = 1$, we recover the uniform distribution. When $a < 1$, we have the case where the firm’s customers are more affluent in the sense that more consumers have high consumption utilities ($\frac{\partial \phi(r)}{\partial r} > 0$). When $a > 1$, the firm faces more consumers with low consumption utilities ($\frac{\partial \phi(r)}{\partial r} < 0$)(see Figure 1).

Figure 1: Probability Density Function of Trapezoid Distribution
With a trapezoid distribution, if the firm adopts PAAP, its optimal price and profit are given by

\[ p_u^1 = \frac{(1-a)c-a+Z}{3(1-a)}, \quad \pi_u^1 = \frac{[3-2a-c(1-a)+Z][2ac-a-2c+Z][3+c-a(1+c)+Z]}{27(1-a)^2}, \] (5)

where \( Z = \sqrt{[a - (1 - a) c]^2 + 3 (1 - a) (1 + ac)} \).

If the firm adopts PAYW, we can use equation (2.2) to derive the firm’s profit as

\[ \pi_p^1 = \frac{\lambda}{6} (1 - c)^2 [2 (2 + c) - a (1 + 2c)]. \]

Before we proceed, define

\[ \lambda^*(c, a) = \frac{2[3 - c - a (2 - c) - Z] [a + 2c - 2ac - Z] [3 + c - a (1 + c) + Z]}{[-9 (1 - a)^2 (1 - c)^2 [4 - a + 2c (1 - a)]]}. \]

A comparison of these two profit functions leads to the following proposition 7.

**Proposition 1** The firm should adopt PAYW when fair-minded consumers are sufficiently generous \((\lambda > \lambda^*(c, a))\), but should adopt PAAP if they are not \((\lambda \leq \lambda^*(c, a))\). Furthermore, \(\lambda^*\) decreases with \(a\) as well as with \(c\) for \(0 \leq a < 1\), but increases with \(c\) for \(1 < a \leq 2\).

The first part of Proposition (1) essentially confirms a very intuitive idea: a firm will adopt PAYW if consumers are sufficiently “fair-minded” or generous in that they are willing to compensate the firm voluntarily with a sufficient portion of the value they and the firm jointly create. Therefore, the success of PAYW will critically hinge on the kind of customers the firm attracts. For that reason, “fans” can be the right crowd for a musician to try PAYW on, but a big oil or big pharma would be ill-advised to try the pricing mechanism.

Proposition 1 also suggests that how consumers’ utilities are distributed, \(a\) in our case, is also an important determinant in a firm’s pricing choice. Here, we find that it is not a higher concentration of high willingness consumers that is more conducive to the adoption of PAYW, but to the contrary,

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6The following apply to cases of \(a \neq 1\). The optimal price and profits of PAAP and profits of PAYW for \(a = 1\) are given in (3.3) and (3.4).

7The proof of all propositions and detailed analysis are provided in the Appendix.
a higher concentration of low willingness to pay consumers that will motivate the firm to use PAYW. In other words, the greater the concentration of the low willingness to pay consumers, the lower the portion of the value that consumers must transfer to the firm to induce it to adopt PAYW. Interestingly, anecdotal evidence seems to support this conclusion that PAYW tends to be observed in markets where there is a concentration of low willingness to pay customers, whether it is for selling music, museum admissions, or food and beverages.

The intuition behind this conclusion is that under PAYW, the firm’s demand is $1 - \Phi(c)$, while under PAAP, it is $1 - \Phi(p^*)$. Given that $p^* > c$, this must mean that the demand enhancing effect of PAYW is stronger when $a$ is larger, as more consumers have low consumption utilities in the neighborhood of the marginal cost $c$, the cutoff point for marginal consumers under PAYW. In other words, a larger $a$ will add more profitable sales as the firm switch from PAAP to PAYW.

This analysis also helps to see that PAAP is a pricing mechanism that is most effective in exploiting high willingness to pay consumers, as only consumers with their willingness to pay higher than the set price will make a purchase. In contrast, PAYW is most effective as a way to penetrate deep into low willingness to pay consumers. This is why a higher concentration of low willingness to pay customers in a market can only enhance the advantage of PAYW over PAAP.

Proposition 1 also shows that the effect of changes in marginal costs on the threshold level of $\lambda$ depends on the level of $a$. As expected, the adoption of PAYW is less likely (a higher $\lambda^*$) if the marginal cost $c$ is larger. However, this happens only when there is a small concentration of high willingness to pay consumers ($a > 1$). When there is a larger concentration of high willingness to pay consumers ($a < 1$), the adoption of PAYW actually becomes more likely (a lower $\lambda^*$) as $c$ increases. In other words, a higher marginal cost gives the firm more, instead of less, incentives

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8Note, that for $a > 1$ PAYW can be more profitable than PAAP even if consumers keep most of the surplus, i.e., $\lambda \leq \frac{1}{2}$.

9Precisely, we have $\Phi(p^*) - \Phi(c) = \Phi(c + \frac{1-\Phi(p^*)}{\phi(p^*)}) - \Phi(c) = \frac{1-\Phi(p^*)}{\phi(p^*)} \phi(p^*)$ according to the mean value theorem where $c < p' < p^*$ and $p' = \frac{c + p^*}{2}$ under the trapezoid distribution. Because $p^*$ decreases with $a$ as more consumers are in the low end of the consumption utility distribution and $\frac{\phi(p')}{\phi(p^*)}$ increases with $a$ as the curve of $\phi(r)$ becomes “steeper” with $a$ increasing, we have $\frac{1-\Phi(p^*)}{\phi(p^*)} \phi(p')$ increases with $a$. 

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to adopt PAYW, when more consumers in the market have high willingness to pay. This perhaps explains why in a high marginal cost industry, like in the restaurant business, a firm like *Around the Corner* can profitably use PAYW. The key is to attract a high concentration of high willingness to pay customers!

Intuitively, when \( a < 1 \), the demand enhancing effect of PAYW becomes stronger when \( c \) increases as more consumers are excluded from buying the product under PAAP than under PAYW. Consequently, the firm is more likely to adopt PAYW when \( a < 1 \) and \( c \) increases. We can see this from the fact that there are more consumers in the high end of the consumption utility distribution so that we have \( \phi(c) < \phi(p^*) \), where \( \phi(c) \) is the marginal demand reduction under PAYW when \( c \) increases, and \( \phi(p^*) \) is the marginal demand reduction under PAAP when \( c \) increases (\( p^* \) increases with \( c \)). In contrast, \( \phi(c) > \phi(p^*) \) when \( a > 1 \) because there are more consumers at the low end of the consumption utility distribution. Therefore, the demand enhancing effect of PAYW becomes weaker when \( c \) increases so that the firm is less likely to adopt it. The above discussion also explains why \( \lambda^* \) is invariant with \( c \) when consumers’ willingness to pay follows a uniform distribution as \( \phi(c) = \phi(p^*) \) in that case.

In summary, the analysis in this section suggests three testable hypotheses. First, when consumers are more fair-minded in that they are willing to let the firm to keep more of the surplus in a transaction, the firm is more likely to adopt PAYW. Second, a higher concentration of low willingness to pay customers are conducive to a firm’s adopting PAYW, all else being equal. Third, a higher marginal cost can incentivize a firm to adopt PAYW if there is a concentration of high willingness to pay consumers.

### 4 Freeloaders and PAYW

In reality, not all consumers are fair minded. The *Radiohead* example shows that a significant fraction of even “fans” are freeloaders, those who do not pay anything for a product when they do not have to. The presence of freeloaders in a market will obviously reduce a firm’s incentive
to adopt PAYW. However, it is not clear to what extent PAYW can tolerate freeloaders before it is dominated by PAAP. In other words, how small does the segment of freeloaders have to be in order for PAYW to benefit a firm? The Radiohead example seems to suggest that PAYW can be profitable even with the presence of a sizable freeloader segment. We will investigate that question in this section by allowing a $\theta$ fraction of consumers in the market to be freeloaders and the rest to be fair minded consumers as described in the previous section.

As a fair-minded consumer with consumption utility $r$ will buy only if $r \geq c$ and pay $c + \lambda(r - c)$ and a freeloader PAYW nothing to consume a product, the firm’s profit under PAYW is given by

$$\pi_p^2 = (1 - \theta) \int_c^1 \lambda(r - c)\phi(r)d - \theta c = \frac{\lambda(1 - \theta)(1 - c)^2[2(2 + c) - a(1 + 2c)]}{6} - \theta c.$$  

Notice that we have $\pi_p^2 = (1 - \theta)\pi_p^1 - \theta c$. The reason is that each freeloader consumes a unit but PAYW nothing, while the firm has to incur the cost of providing the additional unit. Also for that reason, we always have $\frac{\partial \pi_p^2}{\partial \theta} < 0$. Therefore, the firm will adopt PAYW if and only if $\pi_p^2 \geq \pi_p^1$ (see equation (5)), or

$$\theta \leq \frac{1}{6c + \lambda(1 - c)^2[4 - a + 2c(1 - a)]} \left[ \frac{2}{9(1 - a)^2}\right] W + \frac{\lambda(1 - c)^2[4 - a + 2c(1 - a)]}{[3 + c - a(1 + c) + Z]} ,$$  

where $W = (3 - 2a - c + ac - Z)(a + 2c - 2ac - Z) [3 + c - a(1 + c) + Z]$.

This expression is fairly complex. However, it is straightforward to show that the number of freeloaders ($\theta$) and the fraction of surplus that fair-minded consumers are willing to pay the firm ($\lambda$) as the fair compensation are compensatory for each other in that a higher $\theta$ needs to be compensated by a higher $\lambda$ in order for the firm to have the same incentive to choose PAYW. In other words, a small number of freeloaders or a high level of fair compensation are all conducive to the firm’s choosing PAYW. Therefore, it is not surprising that PAYW can be optimal for a firm even though a significant number of freeloaders are present, so long as the fair-minded customers are also very generous customers, as is apparently the case for Radiohead. To further investigate the number of freeloaders as a determinant for PAYW, we can set $\lambda = 1$, the maximum value, and analyze the tradeoff between $\theta$ and $c$. The analysis is summarized in the following proposition.
**Proposition 2** Regardless of how consumers are distributed, i.e. for all \( a \in [0, 2] \), if the number of freeloaders are sufficiently large, PAYW is never the optimal strategy relative to PAAP even if the marginal cost is zero. When PAYW dominates PAAP, a lower marginal cost will allow the firm to tolerate more freeloaders.

Proposition 2 is illustrated in Figure 2. What this proposition suggests is that PAYW is not for every market and hence we should not be surprised by the fact that we do not see a wide application of PAYW in different industries, certainly not in those where there are too many freeloaders and the marginal cost is too high. In the case of *Radiohead*, both factors favored PAYW, given the sizable fan base and zero marginal cost for downloading. As the flip side of the coin, Proposition 2 also establishes that as the optimal strategy, PAYW does not require either zero marginal cost or the absence of freeloaders. Therefore, we should not be surprised, either, by the fact that PAYW is used in markets where a significant number of freeloaders or a substantial marginal cost are present. As illustrated in Figure 2, the maximum proportion of freeloaders in the market can be as high as 40% to 50% if the marginal cost is zero. The proportion can be significantly larger if we had incorporated in our model the benefits of free publicity, consumer good will, and opportunities for cross-selling that may come with the adoption of PAYW. *Radiohead* has apparently taken advantage of all these benefits.

Proposition 2 also suggests, as a managerial insight, that at any given marginal cost, reducing the number of freeloaders is the key to the profitability of PAYW and so is a lower marginal cost at any given proportion of freeloaders in a market. Do this mean that fewer freeloaders are always better for a firm under PAYW? Or asked differently, should a firm always try to discourage freeloaders? The answer is, rather suprisingly, “not always.”

Consider, for instance, a visit to the American Museum of Natural History in New York City. The admission is voluntary and the suggested “donation” is $15. However, depending on whether there is a queue, a visitor may feel more or less generous. In this case, social influence or “social proof” can affect consumers’ sense of fairness and generosity (Cialdini and Goldstein, 2001; Cialdini, 1984). We can model the situation by extending our model and assuming that fair-minded
Figure 2: The effect of freeloaders and marginal costs on the optimal pricing strategy

consumers are affected by the number of consumers in the market. More specifically, we can capture the effect of social influence on the fairness parameter $\lambda$ by assuming that $\lambda$ increases with the demand for the product ($D$). We then have

$$\lambda = \lambda_0 + g(D)(1 - \lambda_0),$$

where $D = (1 - \theta)[1 - \Phi(c)] + \theta$, $\lambda_0$ captures the minimum level of fairness on the part of fair-minded consumers absent of any social influence effect, and $g(D)$ is a function that increases with demand $D$, captures the effect of social influence on consumers’ sense of fairness. When $g(D) \equiv 0$ for all $D$, we go back to the previous model.

The firms’ profits in the presence of a social influence are given by

$$\pi_p^{SI} = (1 - \theta) \int_c^1 \lambda(r - c) \phi(r) dr - \theta c.$$  \hspace{1cm} (6)

It is easy to see $\frac{\partial D}{\partial \theta} \geq 0$, but the effect of an increase in $\theta$ on the firm’s profits is not as obvious. On the one hand, there is a negative effect because the firm must satisfy demand from more consumers
and therefore incur the marginal cost of $c$ per unit while generating no revenues. On the other hand, as $\theta$ increases, fair-minded consumers are willing to pay more to the firm in response to increasing social influence. Therefore, it is possible that the latter effect may dominate the former, leading to higher profits for the firm when $\theta$ increases.

To demonstrate this, assume that $g(D)$ takes a S-shape with regard to $D$,

$$g(D) = k \frac{1}{e^{bL} - e^{-b(D - L)}} \left[1 + e^{bL} \right] \left[1 + e^{-b(D - L)} \right] - \frac{1}{1 + e^{bL}}$$

(7)

where $0 \leq k \leq 1$, $0 \leq L < 1$, $b > 0$. It is easy to verify that $g'(D) > 0$, $g(0) = 0$, $g(1) = k$, and $g(D)$ “takes off” at $L$. A S-shaped $g(D)$ implies that social influence starts to have large effect only if the demand (i.e., the number of consumers buying the product) has reached a critical mass, but such influence saturates with diminishing marginal impact when the demand is very high. Given this specification of $g(D)$, we can show that we can generate $\frac{\partial \pi_{SI}}{\partial \theta} > 0$ (see Appendix).

Therefore, in a market where social influence can elevate consumers’ sense of fairness and generosity, the firm may not be hurt by the presence of more freeloaders. To the contrary, it may benefit from their presence as fair-minded customers may feel more generous to pay.\footnote{Of course, if the presence of more freeloaders will simply influence fair-minded customers not to pay, more freeloaders can never increase a firm’s profitability under PAYW. In practice, it is conceivable that the effect of social influence can go both ways.}

5 Managing Profitability under PAYW

If a firm decides to adopt PAYW as its pricing mechanism, can it do anything further to enhance its profitability? The answer is yes. In the previous section, Proposition 2 already suggests some ways to improve the firm’s profitability under PAYW. Here we discuss two approaches favored by practitioners: PAYW with the minimum price and PAYW with a suggested price.
5.1 PAYW with the Minimum Price

In some cases, a firm may adopt PAYW, but with an enhanced feature of the minimum price: the firm sets the lower bound for the price paid, but allows consumers to pay as much as they wish as long as the payment exceeds this lower bound. For example, the organizers of the 2005 Los-Angeles Human Rights Watch Annual Dinner\(^{11}\) announced that “Sponsorship packages start at $3,000...” Does this minimum price always enhance the firm’s profitability? If it does, how? We investigate these two questions here.

To start, we assume that the firm sets a minimum price, \(p\). With the minimum price, freeloaders (\(\theta\) in size) purchase a unit if and only if their \(r \geq p\), and must pay the minimum price \(p\) when making their purchase. Fair-minded consumers (\(1 - \theta\) in size) are not affected by the minimum price if \(p < c\) as only those with \(r \geq c\) will make a purchase. However, if \(p \geq c\), fair-minded consumers buy only if \(r \geq p\) and pay \(\max(c + \lambda(r - c), p)\). Thus, there are three relevant intervals we need to consider: \(p < c\), \(c \leq p \leq \lambda + (1 - \lambda)c\), and \(p > \lambda + (1 - \lambda)c\).\(^{12}\)

It is easy to see that when \(p < c\) the firm’s profits increases with \(p\), as the buying behavior of fair-minded consumers is not at all affected by the minimum price. Consequently, the firm is better off raising the minimum price toward \(c\). At the other end, when \(p > \lambda + (1 - \lambda)c\), the firm’s profits are weakly dominated by PAAP, as the firm is effectively charging all consumers the same price.\(^{13}\) Thus, the only relevant case for finding the firm’s optimal minimum price is where \(c \leq p \leq \lambda + (1 - \lambda)c\). In this case, the firm’s profit can be written as

\[
\pi^m_p = (1 - \theta) \int_{p-(1-\lambda)c}^{\lambda(r-c)} \phi(r)dr + \]

\[
(1 - \theta)(p - c)[\Phi\left(\frac{p - (1-\lambda)c}{\lambda}\right) - \Phi(p)] + \theta(p - c)[1 - \Phi(p)],
\]

where the first term in the expression is the profits from the fair-minded consumers unconstrained

\(^{11}\)See www.hrwcalifornia.org/south/LAdinner2005/dinner2005.htm

\(^{12}\)\(\lambda + (1 - \lambda)c\) is a cutoff point because \(\max(c + \lambda(r - c), p) = c + \lambda(r - c)\) can occur only if \(p \leq \lambda + (1 - \lambda)c\) as we have \(r \leq 1\).

\(^{13}\)The analysis is available from the authors by request.
by the minimum price (with \( \max[c + \lambda(r - c), \, p] = c + \lambda(r - c) \)), the second term is the profits from the fair-minded consumers constrained by the minimum price (\( \max[c + \lambda(r - c), \, p] = p \)), and the last term is the profits from freeloaders who make a purchase at the minimum price.

Note that PAYW is a special case of PAYW plus the minimum price with \( p \) set at 0. This means that the firm can never do worse with the option of setting the minimum price if the minimum price is set optimally. Under PAYW plus the minimum price, the firm can force high willingness-to-pay freeloaders to pay the minimum price and screen out the rest who are not willing to pay the price and are a drag for the firm’s profitability. In addition, the firm can collect more payments from those fair-minded customers with \( r \in [p, \, \frac{p - (1 - \lambda)c}{\lambda}] \) who would have paid voluntarily a price lower than \( p \). However, the down side is that PAYW plus the minimum price will also screen out the fair-minded customers with \( r \in [c, \, p] \) who would have paid a price higher than the marginal cost \( c \). Therefore, how much setting the minimum price will improve the firm’s profitability will depend on the tradeoffs among these factors.

To further develop our model and to get more intuition, we can analyze the case in which consumption utility follows a uniform distribution \( (a = 1) \)\(^{14}\). In that simpler case, when \( c \leq p \leq \lambda + (1 - \lambda)c \), the firm’s profits are

\[
\pi^{2m}_p = (1 - \theta)\left[\frac{\lambda(1 - c)^2}{2} + \frac{(1 - 2\lambda)(p - c)^2}{2\lambda}\right] + \theta(p - c)(1 - p).
\]  

(9)

The firm’s problem is to choose the minimum price that maximizes this profit expression. The following proposition summarizes our analysis.

**Proposition 3** The firm’s optimal minimum price when PAYW can be optimal comparing to PAAP and consumers’ consumption utilities are distributed uniformly is \( p^* = \frac{\theta\lambda(1-c)}{2\lambda-1+\theta} \). Furthermore, the optimal minimum price increases with the marginal cost \( c \) \( \frac{\partial p^*}{\partial c} > 0 \), increases with the proportion of freeloaders \( \theta \) \( \frac{\partial p^*}{\partial \theta} > 0 \), and decreases with the generosity of fair-minded consumers \( \lambda \) \( \frac{\partial p^*}{\partial \lambda} < 0 \).

Intuitively, PAYW plus the minimum price is essentially a pricing mechanism that allows the

\(^{14}\)Similar results can be obtained for the case where consumers’ valuation is driven from trapezoid distribution.
firm to charge the minimum price to freeloaders and variable prices to fair-minded consumers and it is a hybrid instrument that combines PAAP and PAYW. As the firm has more an incentive to charge high willingness-to-pay freeloaders and screen out the rest when \( c \) or \( \theta \) is larger, it will raise the minimum price as stated in Proposition 3. This incentive is only tampered by the fact that a higher minimum price will also screen out more fair-minded consumers, especially when they become more generous. From this perspective, it is easy to see why we have \( p^* = c \) when \( \theta = 0 \) as expected, for such a price is not binding for any fair-minded consumers. Furthermore, it is easy to show that we have \( p^* \leq \frac{1+c}{2} \), the optimal price under PAAP. The lower minimum price would allow the firm to expand the demand among fair-minded consumers.

5.2 Suggested Price

Another common practice is PAYW with a suggested price: a firm does not provide an explicit lower bound for the price, but does post a suggested price. More specifically, a firm may suggest a price but let consumers pay as much as they wish. This pricing mechanism is designed to affect fair-minded consumers, as a suggested price serves as a reference price for fair-minded consumers. It has no effect on freeloaders, as they will simply disregard the suggestion. As in the previous case, PAYW with a suggested price can only enhance the firm’s profitability relative to what it can get under the simple PAYW. This is because the firm can always choose to set the suggested price in a way that is not binding to fair-minded consumers.

To see how a suggested price may enhance a firm’s profitability, we need to modify our previous model and introduce the purchase decision rules for fair-minded consumers when a suggested price, \( p_s \), is present. We assume that if \( c + \lambda(r - c) > p_s \), fair-minded consumers pay \( c + \lambda(r - c) \); if \( c + \lambda(r - c) \leq p_s < r \), they pay \( p_s \); if \( c \leq r \leq p_s \), we assume that with an equal probability the fair-minded consumers purchase the product and pay \( r \) or do not make any purchase at all;\(^{15}\) if \( r < c \), fair-minded consumers do not purchase as before.

\(^{15}\)This assumption allows for the possibility that fair-minded consumers may give up on purchasing the product if when their reservation prices are lower than the suggested price.
To facilitate comparison with the minimum price case, we analyze the case in which consumption utilities follow a uniform distribution \((a = 1)\). In this case, when \(\frac{p_s - (1 - \lambda)c}{\lambda} \leq 1\), the firm’s profits are given by

\[
\pi_s^1 = (1 - \theta) \left[ \int_{p_s - (1 - \lambda)c}^{1} \lambda (r - c) dr + (p_s - c) \left[ \frac{p_s - (1 - \lambda)c}{\lambda} - p_s \right] + \frac{1}{2} \int_{c}^{p_s} (r - c) dr \right] - \theta c
\]

and when \(\frac{p_s - (1 - \lambda)c}{\lambda} > 1\), the firm’s profits are given by

\[
\pi_s^2 = (1 - \theta) \left[ (p_s - c)(1 - p_s) + \frac{1}{2} \int_{c}^{p_s} (r - c) dr \right] - \theta c
\]

The firm’s problem under PAYW with a suggested price is to choose the optimal suggested price, \(p_s\), that will maximize the profit functions \(\pi_s^1\) and \(\pi_s^2\). The following proposition summarizes our analysis.

**Proposition 4** The firm’s optimal suggested price when consumers’ valuations are distributed uniformly is:
\[
p_s = \begin{cases} 
\frac{2 + c}{3} & \text{if } \lambda \leq \frac{2}{3}, \\
\frac{c}{3} & \text{if } \lambda > \frac{2}{3}.
\end{cases}
\]

The suggested price always increases with the marginal cost \(c\). In addition, a higher price is suggested if fair-minded consumers are not sufficiently generous.

As shown in Proposition 4, the optimal suggested price is independent of \(\theta\). This is expected as the suggested price does not affect the behavior of freeloaders. Proposition 4 also suggests that it can be optimal to set \(p_s = c\) when \(\lambda\) is large enough. When \(\lambda > \frac{2}{3}\), any suggested price larger than \(c\) will cause some fair-minded consumers to drop out of the market and the cost of such drop-outs would be too large relative to any gain from the suggested price effect. To avoid this cost, the firm can set the suggested price at \(c\) so that it does not alter the buying behavior of the fair-minded consumers at all. Then, PAYW with a suggested price is functionally the same as PAYW.

\(16c + \lambda (r - c) \leq p_s\) can occur only if \(\frac{p_s - (1 - \lambda)c}{\lambda} \leq 1\) because \(r \leq 1\).
Although PAYW with the minimum price and PAYW with a suggested price can both enhance a firm’s profitability, they are clearly not equivalent pricing mechanisms. PAYW with the minimum price, as the analysis in this section shows, targets freeloaders, while PAYW with a suggested price focuses on exploiting fair-minded consumers. Then, how should a firm choose between the two? We answer that question in the following proposition.

**Proposition 5** If fair-minded consumers are sufficiently generous ($\lambda > \frac{2}{3}$), the firm is better off adopting PAYW with the minimum price. If they are not sufficiently generous ($\lambda \leq \frac{2}{3}$), PAYW with the minimum price is optimal if $\theta > \max\{1 - 2\lambda, \theta^*(c, \lambda)\}$;\(^{17}\) PAYW with a suggested price is optimal if $\theta \leq \min\{\theta^*(c, \lambda), \frac{(1-c)^2}{4(1+c+e^2)}\}$ and PAAP is optimal if $\frac{(1-c)^2}{4(1+c+e^2)} \leq \theta \leq 1 - 2\lambda$ and $(1-c)^2 \leq 1 - 2\lambda$.

Intuitively, what Proposition 5 suggests is that if the fair-minded consumers are sufficiently generous, the focus of a pricing decision maker should be on getting freeloaders to pay through a minimum price. If the fair-minded consumers are not sufficiently generous, the focus should be still on freeloaders if there are a sufficient number of them in the market. If the number is sufficiently low, the firm can still profitably deploy PAYW by focusing on fair-minded consumers and using a suggested price.

### 6 Competition

Our analysis so far has shown that the PAYW strategy can be optimal for a monopoly firm. When competition is absent, the PAYW strategy can dominate the PAAP strategy primarily because of the fact that PAYW allows a firm to maximally take advantage of the market demand and to price discriminate among heterogenous consumers. However, it is not clear if in a competitive context, the same incentives would motivate a firm to choose PAYW over PAAP. In their pioneering research, Thisse and Vives (1988) show that firms always have an incentive to choose a flexible

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\(^{17}\text{Where } \theta^*(c, \lambda) = \frac{1}{2(1+c+e^2)} \left[2 - c(1 - 2c) - 5\lambda + (4 - 5c)c\lambda + 3\lambda^2(1 - c)^2 + B\right] \text{ and } B = \sqrt{[3c^2 - 2\lambda + 4c\lambda(1 - 2c) + 3\lambda^2(1 - c)^2][3 + \lambda[(-8 + 3\lambda + c(2 - c)(2 - 3\lambda)]]}.\)
pricing policy in a competitive context such that price discrimination by all competing firms is an unique equilibrium. Furthermore, in such an equilibrium, the competing firms are all worse off than if they both choose to set a uniform price, charging all consumers the same price. In this section, we investigate how competition affects a firm’s choice between PAYW and PAAP strategies and whether competing firms are always worse off when choosing PAYW.

To do that, we relax the monopoly assumption and assume that there are two competing firms A and B that are located at the two ends of the Hotelling line bounded between zero and one. Both firms (A and B) incur the same marginal costs per unit \( c \). Consumers are uniformly distributed along the Hotelling line and incur a constant unit transportation costs, \( t \). Thus, a consumer located at \( x \) on the Hotelling line incurs a disutility of \( tx \) if he purchases the product from firm A and \( t(1-x) \) if he purchases the product from firm B. We assume that consumers all have the same consumption utility from the product in the market denoted by \( V \). We further assume that \( V \) is sufficiently large so that the market is fully covered in equilibrium (i.e., all consumers will purchase a unit) and that the consumers are fair-minded as defined before with \( 0 \leq \lambda \leq 1 \).

Competing firms play a two-stage game. In the first stage, each firm chooses one of the two strategies: PAYW (\( P \)) or PAAP (\( U \)). As each retailer has two options for its pricing strategy, there are four subgames: both firms follow the PAAP strategy (\( UU \)), both firms follow the PAYW strategy (\( PP \)), one firm follows the PAYW strategy and the other chooses the PAAP strategy (\( PU \)) and (\( UP \)). In the second stage, prices are set independently or realized depending upon the choices in the first stage.\(^{18}\)

It is straightforward to analyze these four subgames and derive the equilibrium for the competitive pricing policy game\(^{19}\). We summarize our analysis in the following proposition.

---

\(^{18}\)When both firms follow the PAYW (\( PP \)), a consumer that located in \( x \leq \frac{1}{2} \) pays \( c + \lambda(V - tx - c) \) and buys from firm A, and a consumer located at \( x \geq \frac{1}{2} \) pays \( c - \lambda(V - t(1-x) - c) \) and buys from firm B. When one firm (firm A) follows PAYW and the other (firm B) follows PAAP (\( PU \)), a consumer buys from the PAYW firm if \( p_t \geq c \) and pays \( c + \lambda(p_t - c) \). \( p_t \) is the price that makes this consumer indifferent between buying from the two firms if it were a posted price from firm A. More details can be found in the Appendix.

\(^{19}\)The analysis of the four cases are in the Appendix.
Proposition 6 When consumers are sufficiently fair-minded, i.e. \( \lambda \geq \lambda^{*} \), where \( 0 < \lambda^{*} = \frac{4t}{4V - 4c - t} < 1 \), the Pareto dominant equilibrium is for both firms to choose PAYW (PP). If otherwise, both firms choose PAAP (UU). In the equilibrium where firms choose PAYW, they can both be better off than if they were both to choose PAAP. Furthermore, if the market is less differentiated and hence more competitive (a smaller \( t \)), competing firms are more likely to choose PAYW (a smaller \( \lambda^{*} \)).

Proposition 6 offers two interesting insights about PAYW as a pricing mechanism. First, PAYW can help a firm to achieve price discrimination as does location-specific pricing discussed in Thisse and Vives (1988). However, by choosing PAYW, a firm does not acquire the same pricing flexibility that the location-specific pricing policy bestows a firm. Said differently, PAYW does not allow a firm to set any price for a specific location at will, as consumers at different locations decide themselves what to pay. For that reason, competing firms do not always choose to price-discriminate through PAYW in our model, while they do in Thisse and Vives (1988) in a bid to acquire pricing flexibility. Second, PAYW can help firms to achieve price discrimination without intensifying price competition, in contrast to location-specific pricing that always worsens price competition. Indeed, PAYW can moderate price competition, as prices in the market become autonomous and competing firms no longer set any prices. For that reason, firms need not be caught in a Prisoner’s dilemma situation. Also for that reason, the more competitive the marketplace becomes, because of less product differentiation (a smaller \( t \)), the more likely it is for competing firms to adopt PAYW. In a more competitive market, a firm simply has more an incentive to surrender its pricing power to consumers.

From this perspective, it is not surprising that we tend to see PAYW in more competitive industries and during economic downturns.

\(^{20}\)When multiple equilibria exist, we use the Pareto dominance criterion to refine the equilibria
7 Conclusion

We have shown in this paper that given the right conditions, a firm may very well do better letting consumers to decide how much a product is worth and how much they pay to get the product, instead of posting a price itself. PAYW can dominate PAAP as a pricing mechanism because it, first and foremost, helps a firm to achieve maximum market penetration. By letting buyers to determine the prices they pay, the firm has taken away the last obstacle that a consumer faces in making a purchase. Furthermore, PAYW is also an effective way for a firm to implement price discrimination. Traditionally, price discrimination is achieved through either consumer self-selection or firm targeting. PAYW is essentially an autonomous price discrimination mechanism that allows consumers to pay different prices out of their fairness concerns or conscience. This form of price discrimination has the unique advantage of moderating price competition: competing firms no longer set prices so that they cannot help but competing on factors other than price.

The right conditions we have identified are essentially three. First, the existence of fair-minded customers in a market and their sufficient generosity are the necessary conditions for PAYW to be more profitable than PAAP. If all consumers are self-interested and economically rational, PAYW can never be an optimal strategy for selling a product unless, of course, the firm uses PAYW to achieve some other strategic objectives. When these necessary conditions are present, we have shown that the marginal cost of the product needs to be sufficiently small, too, but not necessarily close to zero. A lower marginal cost should allow a firm to tolerate more freeloaders. Second, as PAAP is most effective at exploiting the high end of a demand, while PAYW the low end, it is not surprising that a high concentration of low willingness to pay customers is conducive to a firm adopting PAYW. For the same reason, in a high-end market where there is a high concentration of high willingness to pay customers, a higher marginal cost should give a firm more an incentive to adopt PAYW in place of PAAP. Third, a competitive marketplace is where PAYW can easily dominates PAAP. In the extreme case of perfect competition, for instance, competing firms will charge a price equal to marginal costs under PAAP and make zero profit. However, under PAYW, firms cannot and do not compete on price. They make the profit that fair-minded consumers are
willing to give.

Our analysis has also shown that a firm can improve its profitability under PAYW by imposing a minimum price or posting a suggested price. The minimum price can screen out freeloaders while the suggested price can modify the paying behaviors of fair-minded customers. Indeed, from our model, we can see that anything that a firm can do to encourage consumers to become more fair-minded can also achieve the same objective.

Reference


8 Appendix

Proposition 1: If \( a = 1 \), \( p_u^1 = \frac{1+e}{2} \), \( \pi_u^1 = \frac{(1-e)^2}{4} \) as given in (3.3). If \( a \neq 1 \), we can solve the optimal PAAP price from (2.1) which leads to \( p_u^1 = \frac{(1-a)c-a+Z}{3(1-a)} \) and \( \pi_u^1 = \frac{[3-2a-c(1-a)+Z][2ac-a-2c+Z][3+c-a(1+c)+Z]}{27(1-a)^2} \).

It can be verified that \( p_u^1 \) and \( \pi_u^1 \) converge to their value at \( a = 1 \) when \( a \to 1 \) either from right and left. The firm profits when it adopts the PAWY pricing strategy are \( \pi_p^1 = \frac{\lambda}{6}(1-c)^2 \).

\( \pi \) converge to their value at \( a = 1 \) when \( a \to 1 \) either from right and left. Finally, we can also verify that \( \frac{\pi^*}{\partial a < 0, \frac{\pi^*}{\partial c > 0} \) when \( 0 \leq a < 1 \) and \( \frac{\pi^*}{\partial c > 0} \) when \( 1 < a \leq 2 \).

Proposition 2: \( \pi_p^2 - \pi_u^1 = \frac{[3-2a+c(1-a)+Z][2ac-a-2c+Z][3+c-a(1+c)+Z]}{27(1-a)^2} - c\theta - \frac{\lambda(1-c)^2(1-\theta)[a+2ac-2(2+c)]}{6} \).

When \( \lambda = 1 \), \( \pi_p^2 - \pi_u^1 \) is positive for \( \theta < \theta^* = \frac{6}{(1-c)^2(1+2c)^2} \left[ \frac{1}{6}(1-c)^2[a+2ac-2(2+c)] - \frac{1}{27(1-a)^2} \right] \) when the marginal costs are zero, \( \theta^*(c = 0) = \frac{12L[3+\sqrt{3(a-3)(a-3)}-a-3][21+5a+4\sqrt{3-4(3-1)}]}{9(a-4)(1-a)^2} > 0 \). For values of \( 2 \geq a \geq 0 \) and \( 1 \geq c \geq 0 \) we get that \( \frac{d\theta^*}{dc} < 0 \).

Social-proof: We consider the case where \( \lambda = \lambda_0 + g(D)(1-\lambda_0) \), and \( g(D) = k \frac{1+e^{-b(D-L)}}{e^{b(D-L)-e^{-b(D-L)}}} \). The firms’ profits are given by \( \pi_p^{SI} = (1-\theta)\int_c^1 \lambda(r-c)\phi(r)dr - \theta c \). We can show that it is possible to have \( \frac{\partial\pi_p^{SI}}{\partial \theta} = \frac{1}{6}[6c + [b(a(1-c) + c)(1-c)^2c(a + 2ac - 2(2 + c)]k(1-\theta)(1 - \lambda)] \).

\( e^{b[1-L-[a(1-c)+c(1-\theta)](1 + e^{b(L-1)})+1 + e^{b(L-1)}(1 + e^{b[1-L-a(1-c)+c(1-\theta)]})^2]} \)

\( + (1-c)^2[a+2ac-2(2+c)][k(1-\lambda_0)(1+e^{b(L-1)})(1+e^{-bL})(\frac{1}{1+e^{-bL}}+\frac{1}{1+e^{b[1-L-a(1-c)+c(1-\theta)]}})(e^{-bL}-e^{b(L-1)})] > 0 \). For example, consider the case where \( a = 1.8, \lambda_0 = 0.25, k = 0.6, \theta = 0.13, c = \)
0.2L = 0.45 and b = 1.3. In such case \( \frac{\partial \pi_{Sf}}{\partial b} = 0.2 > 0 \).

**Proposition 3:** In order to find the optimal minimum price we derive the profits \( \pi_{2m}^p = (1 - \theta) \left[ \frac{\lambda(1-c)^2}{2} + \frac{(1-2\lambda)(p-c)^2}{2\lambda} \right] + \theta(p-c)(1-p) \) with respect to price, \( p \). This first order condition yields \( p^* = \frac{\theta\lambda - (1-\theta - 2\lambda + \theta)\lambda c}{1-\theta - 2\lambda} \). Note that \( p^* > c \) for values of \( 1 - \theta - 2\lambda < 0 \). For PAYW with minimum price to be more profitable than PAAP, we must have \( \lambda > \frac{1}{3} \). This is because (i) \( \pi_{2m}^p \) decreases with \( \theta \), (ii) \( p^* = c \) when \( \theta = 0 \) which means that PAYW with minimum price is the same as PAYW without minimum price at \( \theta = 0 \), and (iii) \( \lambda > \frac{1}{3} \) is required for PAYW to be more profitable than PAAP when \( a = 1 \) and \( \theta = 0 \) as shown by (3.3) and (3.4) in the paper. Because \( 1 - \theta - 2\lambda < 0 \) always true when \( \lambda > \frac{1}{3} \), \( p^* > c \) always holds when PAYW with minimum price is more profitable than PAAP. Deriving the optimal minimum price with respect to \( c \) yields \( \frac{\partial p^*}{\partial c} = 1 + \frac{\theta \lambda}{1-\theta - 2\lambda} \), which is positive for \( \theta > 0 \) when \( \lambda > \frac{1}{2} \). Deriving the optimal price with respect to \( \theta \) yields \( \frac{\partial p^*}{\partial \theta} = \frac{\lambda(1-c-2\lambda + 2\lambda c)}{(1-\theta - 2\lambda)^2} = -\frac{\lambda(1-c)(1-2\lambda)}{(1-\theta - 2\lambda)^2} \), which is always positive for \( \lambda > \frac{1}{2} \). Deriving the optimal price with respect to \( \lambda \) yields \( \frac{\partial p^*}{\partial \lambda} = -\frac{\theta(1-c)(1-\theta)}{(1-\theta - 2\lambda)^2} \), which is always negative for \( c < 1 \) and \( \theta < 1 \).

**Proposition 4:** From the expression \( \pi_{1s}^p \), it can be easily seen that \( p_{1s}^* = c \) and \( \pi_{1s}^p (p_{1s}^*) = -c\theta + \frac{\lambda(1-c)^2(1-\theta)}{2} \) if \( \lambda > \frac{2}{3} \); and \( p_{1s}^* = \lambda + (1-\lambda)c \) if \( \lambda \leq \frac{2}{3} \) (which is the maximum possible \( p \) satisfying \( p_{1s}^* - \theta(1-\lambda)c \leq 1 \)). The first order condition of \( \pi_{2s}^p \) with respect to \( p_s \), \( \frac{\partial \pi_{2s}^p}{\partial p_s} \), yields \( p_{2s}^* = \frac{2+c}{3} \) and \( \pi_{2s}^p (p_{2s}^*) = \frac{(1-c)^2 - \theta(1+c+c^2)}{3} \). Because \( \pi_{2s}^p (p_{2s}^*) - \pi_{1s}^p (p_{1s}^*) \leq 0 \) when \( p_{1s}^* = \lambda + (1-\lambda)c \) and \( \pi_{2s}^p (p_{2s}^*) - \pi_{1s}^p (p_{1s}^*) = c \) \( \leq 0 \) for \( \lambda > \frac{2}{3} \), the optimal suggested price is given by \( p_s = \frac{2+c}{3} \) if \( \lambda \leq \frac{2}{3} \) and \( p_s = c \) if \( \lambda > \frac{2}{3} \). Note that \( \frac{2+c}{3} - c = \frac{2(1-c)}{3} > 0 \).

**Proposition 5:** The firm profits when it adopts the PAYW with minimum price are \( \pi_{2m}^p = \frac{(1-c)^2 \lambda (1-2\theta(1-\lambda) - 2\lambda)}{2(1-\theta - 2\lambda)} \), the firms profits when it adopts the PAYW with suggested price are \( \pi_{1s}^p = \frac{1}{2}(1-\theta)(1-c)^2 - c\theta \), for \( \lambda > \frac{2}{3} \) and \( \pi_{2s}^p = \frac{1}{2}[1 - \theta - c[2 - c + \theta(1+c)] \), for \( \lambda \leq \frac{2}{3} \), the firm profits when it adopts the PAAP pricing strategy is \( \pi_u = \frac{(1-c)^2}{4} \).
For values of $\lambda > \frac{2}{3}$, \(\pi_p^{2m} - \pi_p^{1s} = \frac{\theta[4c+4(1-c)\theta-2c(1-\theta)]}{2(1+\theta+2\lambda)}\) which is positive for all values as \(\theta > 0 > \frac{c(2-4\lambda)}{2c+\lambda(1-c)^2}\). \(\pi_p^{2m} - \pi_p^1 = \frac{(1-c)^2(1-\theta)(1-2\lambda)^2}{4(1+\theta+2\lambda)}\) which is always positive for \(\theta > 1 - 2\lambda\) (as \(1 - 2\lambda < 0\) for \(\lambda > \frac{1}{2}\)).

For values of \(\lambda \leq \frac{2}{3}\), \(\pi_p^{2m} - \pi_p^{2s} = \frac{[\theta-1+c(2\theta-c(1-\theta))]^2}{3} + \frac{(1-c)^2\lambda(1-2\lambda-\theta)(1-\lambda)}{2(1-\theta-2\lambda)}\) which is positive for values of \(\theta > \theta^*(c, \lambda)\). \(\pi_p^{2m} - \pi_p^1 = \frac{(1-c)^2(1-\theta)(1-2\lambda)^2}{4(1+\theta+2\lambda)}\) which is positive for \(\theta > 1 - 2\lambda\).

\(\pi_p^{2s} - \pi_p^1 = \frac{[(1-c)^2-4\theta(1+c+c^2)]}{12}\) which is positive for \(\theta < \frac{(1-c)^2}{4(1+c+c^2)}\).

**Proposition 6:** We start with the analysis of the scenario where both firms follow the PAAP strategy (UU). Assume \(V\) is sufficiently large so that the market is always covered. In this case consumer located in \(x\) gains surplus of \(V - tx - p_1\) if buying the product from firm A and gains surplus of \(V - t(1-x) - p_2\) if buying the product from firm B. Firms’ A and B profits are \(\pi_{uu}^A = (p-c)x\) and \(\pi_{uu}^B = (p-c)(1-x)\) respectively, where \(x\) is the location of a consumer that is indiff erent between purchasing a unit from firm A or from firm B. It is easy to show that in equilibrium, the price and the firms’ profits are given by \(p_{uu} = c + t, \pi_{uu}^A = \pi_{uu}^B = \frac{t}{2}\).

Next, we analyze the scenario where both firms follow PAYW strategy (PP): In this case a consumer located in \(x < \frac{1}{2}\) will purchase from firm A as the consumption utility from firm A’s product, \(V - tx\), is higher that that from firm B’s product, which is \(V - t(1-x)\). This consumer will then pay \(c + \lambda(V - tx - c)\) to firm A. Similarly, a consumer located at \(x > \frac{1}{2}\) will buy from firm B and pays \(c + \lambda(V - t(1-x) - c)\) to firm B. Therefore, firms’ profits are \(\pi_{pp}^A = \lambda \int_0^{\frac{1}{2}} (V - tx - c)dx\) and \(\pi_{pp}^B = \lambda \int_{\frac{1}{2}}^1 [V - t(1-x) - c]dx\) respectively. It is easy to show that \(\pi_{pp}^A = \pi_{pp}^B = \frac{\lambda(V-t-4c)}{8}\).

Finally, we analyze the scenario where firm A follows the PAYW strategy and firm B follows the PAAP (PU). Denote a consumer’s maximum willingness to pay to firm A given firm B’s price as \(p_t\). \(p_t\) is the price that makes the consumer indifferent between buying from firm A and firm B, which is determined by \(V - tx - p_t = V - t(1-x) - p_2\) and results in \(p_t = t(1-2x) + p_2\). We assume that a consumer will buy from firm A if and only if \(p_t \geq c\) and she will pay \(c + \lambda(p_t - c)\) if he buys from firm A. This implies that a consumer will buy from firm A if the maximum surplus he can get from firm A and still being fair (i.e., paying at lease \(c\)) is higher than the surplus she can get from firm B. Then she will give a \(\lambda\) portion of the maximum total surplus from the transcation, \(p_t - c\), to firm A if she buys from it. From \(p_t \geq c\), we have \(x \leq \frac{t-c+p_2}{2t}\). Thus, consumers with \(x \leq \frac{t-c+p_2}{2t}\) buys from firm A while the others buy from firm B. Firms’ A and
profits are then given by $$\pi^A_{pu} = \int_0^{t-c+p_2} \lambda (p_t - c) dx = \lambda \int_0^{t-c+p_2} [t (1 - 2x) + p_2 - c] dx$$ and $$\pi^B_{pu} = (p_2 - c) \left( 1 - \frac{t-c+p_2}{2t} \right)$$. Solving for the optimal price of firm B, we obtain $$p_2 = c + \frac{t}{2}$$.

Consequently, firms A and B profits are given by $$\pi^A_{pu} = \frac{9\lambda}{16}$$ and $$\pi^B_{up} = \frac{t}{8}$$ respectively.

Because $$\pi^A_{uu} \geq \pi^A_{pu} \rightarrow \lambda \leq \frac{8}{9}, \pi^B_{pp} \geq \pi^B_{pu} \rightarrow \lambda \geq \frac{t}{4V-4c-t}$$, and $$\frac{t}{4V-4c-t} < \frac{8}{9}$$ as $$V > 2t + c$$ is required for the full market coverage under $$UU$$ case, we have $$UU$$ to be equilibrium if $$\lambda \leq \frac{8}{9}$$ and $$PP$$ to be equilibrium if $$\lambda \geq \frac{t}{4V-4c-t}$$. For $$PP$$ to be equilibrium of Pareto dominance, we need $$\pi_{pp} > \pi_{uu} \rightarrow \lambda > \frac{4t}{4V-4c-t}$$. Also, $$\frac{4t}{4V-4c-t} < \frac{8}{9}$$ as $$V > 2t + c$$. Therefore, $$PP$$ is the Pareto dominance equilibrium if $$\lambda > \lambda^* = \frac{4t}{4V-4c-t}$$ and $$PP$$ is the unique equilibrium if $$\lambda > \frac{8}{9}$$. Note that $$\frac{\partial \lambda^*}{\partial t} = \frac{16(V-c)}{(4V-4c-t)^2} > 0$$. 

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