From private banking to central banking: ingredients of a welfare analysis*

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June 16, 2004

Abstract

An earlier analysis that compares allocations achievable using inside (private) money to those achievable using outside (government) money is extended. It is shown that outside-money allocations are a subset of inside-money allocations provided that the inside money issued by one issuer can be distinguished from that issued by others. If that recognizability assumption is weakened, then the subset result could conceivably be over-turned. Even so, the analysis suggests that the outside-money arrangement, with its uniform money, should be managed so as to attain some of the benefits of inside money.

JEL classification: E 40

Key words: inside money, outside money, monitoring, matching model

1 Introduction

Central banks, at least in the U.K. and the U.S., emerged as monopoly issuers of banknotes from systems in which there were many private banks issuing

*This paper was written during a sabbatical leave from The Pennsylvania State University. Parts of that leave were spent as a visiting scholar at the reseach departments of the Federal Reserve Banks of Cleveland, Kansas City, and Minneapolis. I am indebted to those departments for providing research support and stimulating research environments. Needless to say, the views expressed are my own and not necessarily those of the above Banks or the Federal Reserve System. I am also indebted to Alexei Deviatov, Jim Jordan, and Ed Nosal for comments on earlier drafts.

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banknotes. The emergence was accompanied by debates about how private banks should be regulated and about how central banks should behave. In the U.K., the debates were between the *banking* and *currency* schools, with the former advocating some version of laissez-faire in intermediation and the latter advocating some version of hard money or what much later came to be called monetarism. In the U.S., the issues were similar, but advocates of the banking school position tended to be labeled advocates of the *real-bills doctrine*. In this paper, I revisit those issues by setting out a framework in which an arrangement that resembles private banknote issue can be compared with one that resembles a central bank monopoly on note issue.

My starting point is the model that Cavalcanti and I used to compare inside and outside money (see [2] and [3]). We adopted the pairwise-meeting setting in [12] and [14] and assumed that some people are (perfectly) monitored, while the rest are not monitored at all. For well-known reasons (see, for example, [7]), the non monitored people are the source for a demand for payable-to-the-bearer, tangible media of exchange. Here, I also make those assumptions, but depart from the Cavalcanti and Wallace analysis in the following ways.

First, I add a meeting with the planner that alternates in time with the pairwise meetings of [12] and [14]. The meeting with the planner, which can be interpreted as a meeting with the central bank, is used solely for risk sharing; no production or consumption occurs and, therefore, no utility is realized. Second, I allow the non monitored people to hide money. Third, I let individual money holdings be general, instead of restricting such holdings to be in the set $\{0, 1\}$. Finally, and, most significantly, I make different assumptions regarding the kind of threats that are allowed.

In Cavalcanti and Wallace, monitored people who defect were punished by permanent autarky. In contrast, I assume that any such defector joins the ranks of the non monitored people and suffers no further punishment. I also assume that defection by a single person does not give rise to punishment of a significant portion of the entire economy and that people can distinguish the money issued by one monitored person from that issued by other monitored people. The last assumption, which I label the recognizability assumption, allows the money issued by a defector to be rejected without shutting down a significant portion of the entire economy.

Under those assumptions, I show that inside money has the same kind of advantage over outside money that Cavalcanti and I found: the set of outside-money, incentive-feasible allocations is a subset of the set of inside-money, incentive-feasible allocations. However, as will be explained in detail later, the source of the possibility of achieving better allocations under inside money is different from what it was in Cavalcanti and Wallace.

After presenting the subset result, I discuss whether the recognizability assumption is reasonable and speculate about the consequences of abandoning it. One consequence of weakening it, I suggest, is greater vulnerability of inside-money to counterfeiting. That greater vulnerability could offset the advantage that inside money has when the recognizability assumption holds. However, even if it does, the features of the model that give rise to that advantage have implications for the kind of risk-sharing that should occur in the meeting with the planner under outside-money—under a central-bank monopoly on note issue.

One qualification should be noted at the outset. Two questions arise in any model of money. Why are people using money when trades could conceivably be accomplished with some version of borrowing and lending, and why are people using money when trades could conceivably be accomplished by trading assets with higher rates of return? Here I deal only with the first. I avoid the second troublesome question by assuming that all real objects are perishable. We will see that quite a bit can be said about inside and outside money without dealing with it.

2 A model

As noted above, the background environment is borrowed from [12] and [14]. Time is discrete. There is a non-atomic, unit measure set of each of $K \geq 3$ specialization types of infinitely lived people and there are K distinct, produced, and perishable goods at each date. A specialization-type k person, $k \in \{1, 2, ..., K\}$, produces only good k and consumes only good $k+1$ (modulo K). Each person maximizes expected discounted utility with discount factor $\beta \in (0, 1)$. For a specialization-type k person, utility in a period is $u(q_{k+1}) - q_k$, where $q_{k+1} \in \mathbb{R}_+$ is consumption of good $k+1$ and $q_k \in \mathbb{R}_+$ is production of good k. The utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly concave, strictly increasing, continuously differentiable and satisfies $u(0) = 0$ and $u'(\infty) = 0$. In addition, $u'(0)$ is sufficiently large.

Each period is divided into two parts, which can be labeled morning and afternoon. The morning is reserved for random pairwise meetings. A meeting between specialization types k and $k+1$ is called a single-coincidence meeting.

Other meetings are called no-coincidence meetings.[1] In the afternoon, people meet the planner.

The following monitoring assumption is borrowed from [2]. The set of each specialization type is partitioned in an exogenous way into two sets: the fraction m_1 are monitored and the fraction m_2 (with $m_1 + m_2 = 1$) are not. That is, the history of each monitored person is common knowledge, while that of each non monitored person is private to the person. It is as if each monitored person wears a computer chip that transmits everything that the person does to everyone else. In contrast, the only thing known about a non monitored person is the person's type. In particular, a non-monitored person can hide money, but a monitored person cannot.

Finally, to permit a discussion of private money, each person has a printing press capable of turning out indivisible and identical durable objects. The items turned out by the printing press of any one person are distinguishable from those turned out by other peoples' printing presses and from the planner's (central-bank) money.

There are two related economic problems in this model. First, in order to induce a person to produce enough, the person must see a future reward from doing so. Second, from an ex ante point of view, it would be desirable to limit the dependence of a person's current actions, both production and consumption, on the random realizations that the person has experienced.

2.1 A class of symmetric allocations

Throughout, I consider only a limited class of allocations. First, as is fairly standard in the kind of model set out above, nothing depends on specialization type. Therefore, in what follows type will be used to designate only whether the person is monitored: 1 is for a monitored person, 2 is for a non monitored person. Second, the allocations are designed to permit a comparison of two rather special arrangements: either all valuable money is uniform central-bank money—which, perhaps, misleadingly, I label outside money; or there is, in addition, valuable money issued solely by type-1 people—an arrangement I label inside money. Moreover, in the inside-money arrange-

[1] Some economists have expressed concern about the assumption that meetings are random That concern is misplaced. First, the role of such randomness is to generate random earning opportunities and random consumption opportunities. Such randomness could be generated by preference shocks. Second, settings in which every meeting is a single-coincidence meeting have similar implications (see Wallace and Zhu [15]).

ment, I restrict my attention to allocations consistent with all the valuable monies being perfect substitutes and with no type-1 person beginning either a morning or an afternoon with anything other than the person's printing press.

Given that limited purpose, I assume that a person's state, in addition to the person's type, is an element in the set of non negative integers, denoted $\mathbb{Z}$.[2] For a type-2 (non-monitored) person, $z \in \mathbb{Z}$ is the amount of valuable money held. For a type-1 (monitored) person, z has the same interpretation in the outside-money arrangement, but is simply a label in the inside-money arrangement. In either case, for a type-1 person, z is assumed to be publicly known.[3] I let $\mathbb{I} = \{1, 2\}$, the set of types, and let $\mathbb{S} = (\mathbb{I} \times \mathbb{Z})$. An allocation describes what happens in pairwise meetings and what happens in the meeting with the planner as functions of type, state, and date.

I start with pairwise meetings. For reasons described below, I assume that nothing happens in no-coincidence meetings. As regards single-coincidence meetings, the state of a meeting is $(s_c, s_p) \in \mathbb{S}^2$, where the first component describes the consumer and the second the producer as they enter the meeting. Although the discreteness of the set $\mathbb{Z}$ implies that there is a role for lotteries, for what is done here, adding lotteries seems not to matter. Therefore, to keep the notation simple, I assume that both output and state transitions are deterministic. I let $y_t(s_c, s_p)$ denote date-t output, $c_t(s_c, s_p)$ the date-t end-of-meeting state for the consumer, and $p_t(s_c, s_p)$ the date-t end-of-meeting state for the producer. That is, $y_t : \mathbb{S}^2 \rightarrow \mathbb{R}_+$ is date-t output and $c_t : \mathbb{S}^2 \rightarrow \mathbb{Z}$ and $p_t : \mathbb{S}^2 \rightarrow \mathbb{Z}$ are the date-t, end-of-meeting states for consumers and producers, respectively.

In the meeting with the planner, there are new states that depend for each person only on the state of the person. Thus, I let $h_t : \mathbb{S} \rightarrow \mathbb{Z}$ denote that new end-of-afternoon state. The state transitions in the meeting with the planner would seem to make superfluous the state transitions that could conceivably happen in no-coincidence meetings. In no-coincidence meetings, type-2 people could be given money by type-1 people, but it would seem equivalent to have that happen in the meeting with the planner.

[2] I work with a discrete set of individual states solely in order to avoid dealing with measure-theoretic concepts.

[3] In the case of inside money, the allocation determines the interpration of the set $\mathbb{Z}$ for type 1 people. For example, it could be total previous note issue by the person, or it could be the maximum of 0 and total notes destroyed minus total notes issued—a kind of net worth of the person.

Letting $x_t \equiv (y_t, c_t, p_t, h_t)$, an allocation is a sequence $\{x_t\}_{t=0}^{\infty}$. It describes what happens in the economy in the following sense. Given a date 0 probability distribution over $\mathbb{I} \times \mathbb{Z}$, the assumption that pairwise meetings are random determines a distribution over the kinds of meetings that occur. Then, x_0 describes what happens in those meetings and in the subsequent meeting with the planner, and implies a date-1 probability distribution over $\mathbb{I} \times \mathbb{Z}$; and so on.

In order to describe that evolution precisely, let $\pi_t = (\pi_t^1, \pi_t^2)$, where $\pi_t^i : \mathbb{Z} \to [0,1]$ and $\pi_t^i(z)$ is the pre-morning date-t fraction of type-i people in state z, and let $\theta_t = (\theta_t^1, \theta_t^2)$, where $\theta_t^i : \mathbb{Z} \to [0,1]$ and $\theta_t^i(z)$ is the pre-afternoon date-t fraction of type-i people in state z. Then,

$$\theta_t^i = \pi_t^i T_t^i, \tag{1}$$

where the entry in row j and column j' in the matrix T_t^i, the probability for a type i person of leaving a date-t meeting in state j' given that the person was in state j entering the meeting, is given by

$$\begin{aligned} T_t^i(j,j') \;&=\; \frac{K-2}{K}\delta_{jj'} + \frac{m_1}{K}\sum_z \pi_t^1(z)[c_t(j';i,j,1,z) + p_t(j';1,z,i,j)] \\ &\quad + \frac{m_2}{K}\sum_z \pi_t^2(z)[c_t(j';i,j,2,z) + p_t(j';2,z,i,j)]. \end{aligned} \tag{2}$$

Here, $\delta_{jj'} = 1$ if $j = j'$ and 0 otherwise and, in an abuse of notation, $c_t(j'; s_c, s_p) = 1$ if $c_t(s_c, s_p) = j'$ and 0 otherwise, and similarly for $p_t(j'; 1, z, i, j)$. And

$$\pi_{t+1}^i = \theta_t^i H_t^i, \tag{3}$$

where the entry in row j and column j' of the matrix H_t^i is given by

$$H_t^i(j,j') = h_t(j'; i, j), \tag{4}$$

where $h_t(j'; i, j) = 1$ if $h_t(i,j) = j'$ and 0 otherwise.

It is also convenient to define expected discounted utilities in terms of an allocation. First, let $r_t^i : \mathbb{Z} \to \mathbb{R}$, where $r_t^i(j)$, the date-t morning "return" to a type-i person in state j. We have

$$\begin{aligned} r_t^i(j) \;&=\; \frac{m_1}{K}\sum_z \pi_t^1(z)\{u[y_t(i,j,1,z)] - y_t(1,z,i,j)\} + \\ &\quad \frac{m_2}{K}\sum_z \pi_t^2(z)\{u[y_t(i,j,2,z)] - y_t(2,z,i,j)\}. \end{aligned} \tag{5}$$

Let $v_t = (v_t^1, v_t^2)$, where $v_t^i : \mathbb{Z} \to \mathbb{R}$ and $v_t^i(z)$ is the date-t pre-morning discounted expected utility for a type i person in state z, and let $w_t = (w_t^1, w_t^2)$, where $w_t^i : \mathbb{Z} \to \mathbb{R}$ and $w_t^i(z)$ is the date-t pre-afternoon discounted expected utility for a type i person in state z. We have

$$v_t^i = r_t^i + T_t^i w_t^i \text{ and } w_t^i = \beta H_t^i v_{t+1}^i. \tag{6}$$

2.2 Incentive-feasible allocations

The heart of the model is the description of the subset of allocations that are incentive feasible. For type-2 who are not monitored, the constraints—truth-telling and participation constraints—are what they would be in a static model. As regards type-1 people, I assume, as noted above, that each such person has the option to disappear into the ranks of the type-2 people at any time. It is as if each such person can at any time throw away or disable the device that monitors the person. In what follows, I take it for granted that a type-1 person who defects would want to join the ranks of the type-2 people: If the person continued to be monitored, then harsher punishment could be inflicted on the person.

The sequence of actions is as follows. In a single-coincidence meeting at date t, the two people see each other's type in the set $\mathbb{I}$. In addition, the states in $\mathbb{Z}$ of all the type-1 people are seen. Then the people who are type-2 people in the meeting report their money holdings —simultaneously if there are two of them. The only lying that is permitted is under-representation of holdings.[4] Then the allocation—in particular, y_t, c_t, and p_t—has a suggested outcome for the meeting. Then the consumer and producer simultaneously say *yes* or *no* to the suggested outcome. If either says *no*, then both people leave the meeting with no output produced and with no state transitions. Moreover, if a type-1 person says *no*, then that is a defection that converts the person permanently into a type-2 person starting in the afternoon. If a type-2 person says *no*, then that has no future consequences beyond autarky in the current meeting. In addition, because I am focussing on allocations in which type-1 people do not hold money, a type-1 person can defect from an inside-money allocation after trading by failing to destroy any money received in a trade, a failure that is assumed to be publicly observable. The afternoon is similar and simpler because each person is dealt with individually. A

[4]The inability to over-state money holdings gives meaning to the notion that money is tangible.

type-1 person gets to say *yes* or *no* to the suggested state transition: saying *no* leaves the person's state unchanged, but is a defection that converts the person permanently into a type-2 person starting the next morning. A type-2 person reports the person's state in $\mathbb{Z}$, again with only under-representation possible. Then the person gets to say *yes* or *no* to the suggested state transition. As in the morning, saying *no* for a type- 2 person has no future consequences.

Before I express the above conditions in terms of allocations, I should make two comments. First, the above scheme permits only individual defections in pairwise meetings. I would prefer to allow all cooperative pairwise defections. However, that calls for dealing with the pairwise core under asymmetric information, a task which I am not prepared to undertake here.[5] Second, I allow only limited punishments (threats). I rule out all punishments that involve positive-measure responses to defection by a single person. Thus, for example, reversion to autarky for the entire economy is not an allowable punishment for individual defection. However, as spelled out below, in inside-money arrangements, defection by a type-1 person is accompanied by making worthless the money that that person can print.

To express the truth-telling and participation constraints compactly, another bit of intermediate notation is helpful. Let

$$G_t^c(i, z, s) = u[y_t(i, z, s)] + w_t^i[c_t(i, z, s)], \tag{7}$$

the date-t morning payoff to a type i, state z consumer in a meeting with an s producer, and let

$$G_t^p(s, i, z) = -y_t(s, i, z) + w_t^i[p_t(s, i, z)], \tag{8}$$

the date-t morning payoff to a type i, state z producer in a meeting with an s consumer.

The constraints that pertain to type-2 people are identical for outside- and inside-money allocations. Those that pertain to the monitored people are different. Therefore, I give the former first and then give the latter separately for each kind of allocation.

[5]Notice, however, that prevention of over-issue of inside money in meetings between type-1 consumers and type-2 producers is not dependent on ruling out cooperative defections. Such a producer would not be tempted to produce a great deal in exchange for a lot of money if the producer knows that such a trade is a defection that will render the money obtained useless.

The truth-telling constraints for type-2 people in pairwise meetings with other type-2 people are

$$\max_{z' \leq j} \sum_{z} \pi_t^2(z) G_t^c(2, z', 2, z)] = \sum_{z} \pi_t^2(z) G_t^c(2, j, 2, z), \tag{9}$$

and

$$\max_{z' \leq j} \sum_{z} \pi_t^2(z) G_t^p(2, z, 2, z') = \sum_{z} \pi_t^2(z) G_t^p(2, z, 2, j), \tag{10}$$

where the first is for a consumer and the second is for a producer. These must hold for all $j \in \mathbb{Z}$. The participation constraints for these meetings are

$$\min\{G_t^c(2, j, 2, z), G_t^p(2, z, 2, j)\} \geq w_t^2(j), \tag{11}$$

which must hold for all $(j, z) \in \mathbb{Z} \times \mathbb{Z}$. The constraints for type-2 people in meetings with type-1 people can be written

$$\max_{z' \leq j} G_t^c(2, z', 1, z) = G_t^c(2, j, 1, z)] \geq w_t^2(j) \tag{12}$$

and

$$\max_{z' \leq j} G_t^p(1, z, 2, z') = G_t^p(1, z, 2, j) \geq w_t^2(j) \tag{13}$$

where, in each case, the equality is truth-telling and the inequality is the participation constraint. These, also, must hold for all $(j, z) \in \mathbb{Z} \times \mathbb{Z}$. For the centralized meeting, we have

$$\max_{z \leq j} v_{t+1}^2[h_t(2, z)] = v_{t+1}^2[h_t(2, j)] \geq v_{t+1}^2(j) \tag{14}$$

for all $j \in \mathbb{Z}$.

Now the definitions can be given.

Definition 1 *A sequence $\{y_t, c_t, p_t, h_t\}_{t=0}^{\infty}$ is an incentive-feasible, inside-money allocation if there exists $\{v_t, w_t, \theta_t, \pi_t\}_{t=0}^{\infty}$ such that (1)-(14) hold and such that*

$$\min\{G_t^c(1, z, s), G_t^p(s, 1, z)\} \geq w_t^2(0) \text{ for all } (s, z) \in \mathbb{S} \times \mathbb{Z}, \tag{15}$$

$$w_t^1(z) \geq w_t^2(0) \text{ and } v_{t+1}^1[h_t(1, z)] \geq v_{t+1}^2(0) \text{ for all } z \in \mathbb{Z}, \tag{16}$$

$$c_t(2, z, 2, z') + p_t(2, z, 2, z') = z + z' \text{ for all } (z, z') \in \mathbb{Z} \times \mathbb{Z}, \tag{17}$$

and

$$w_t^1[p_t(2, z, 1, z')] \geq w_t^2[z - c_t(2, z, 1, z')] \text{ for all } (z, z') \in \mathbb{Z} \times \mathbb{Z}. \tag{18}$$

Definition 2 *A sequence* $\{y_t, c_t, p_t, h_t\}_{t=0}^{\infty}$ *is an incentive-feasible, outside-money allocation if there exists* $\{v_t, w_t, \theta_t, \pi_t\}_{t=0}^{\infty}$ *such that (1)-(14) hold and such that*

$$\min\{G_t^c(1, z, s), G_t^p(s, 1, z)\} \geq w_t^2(z) \text{ for all } (s, z) \in \mathbb{S} \times \mathbb{Z}, \tag{19}$$

$$w_t^1(z) \geq w_t^2(z) \text{ and } v_{t+1}^1[h_t(1, z)] \geq v_{t+1}^2(z) \text{ for all } z \in \mathbb{Z}, \tag{20}$$

and

$$c_t(s_c, s_p) + p_t(s_c, s_p) = z + z' \text{ for all } (s_c, s_p) \in \mathbb{S} \times \mathbb{S}. \tag{21}$$

Conditions (15) and (19) require that a type-1 person say *yes* to the trade the allocation specifies. Under inside money (see (15)), if the person says *no*, then the person begins the afternoon as a type-2 person with no money. That is the alternative because saying *no* implies no trade, because the type-1 person entered the meeting with no other money (with only a printing press and a label), and because the defector's printing press becomes useless once the person defects. The interpretation is that the money turned out by that press is no longer valued: no one accepts it.[6] Under outside money (see (19)), if the person says *no*, then the person begins the afternoon as a type-2 person with the outside money brought into the meeting. (Notice that the ability of type-2 people to under-represent money holdings implies that v_t^2 and w_t^2 are weakly increasing. Moreover, unless money is worthless, both functions are strictly increasing.) Conditions (16) and (20) require that a type-1 person not defect just prior to the meeting with the planner and that the person says *yes* to the afternoon state transition. Again, they differ regarding the defector's pay-off. Condition (17) says that money holdings are preserved in meetings among type-2 people, while constraint (21) is identical except that it applies to all meetings, not just those between type-2 people.

Condition (18) for inside-money allocations has no analog for outside-money allocations. It says that the type-1 person is willing to destroy any money received in trade with a type-2 person. This is written only for the case when the trading partner is a consumer because if w_t^2 is strictly increasing, then a type-2 producer does not surrender money.[7] (A desirable allocation could call for type-2 people to surrender money to type-1 people when they

[6]Of course, this may hurt those who are holding that money at the time of the defection. If the notes were labeled with issue date, then the printing press could be rendered useless without affecting the notes issued earlier.

[7]It should be understood that condition (18) is void if $z - c(2, z, 1, j) < 0$.

receive output even though the money is to be immediately destroyed. Taking money from type-2 people in such trades can enhance its value and that can, in turn, weaken the constraints involving production by type-2 people.)

For both inside and outside-money allocations, I permit creation of money in the meeting with the planner. If v_t^2 is strictly increasing, then all state transitions for type-2 people have non negative transfers because any type-2 person can always report state 0. More generally, in order to induce truth-telling all transfers to type-2 people must be weakly increasing in a type-2 person's state.[8]

3 A subset result

I can now prove that the set of incentive-feasible outside-money allocations is a subset of the incentive-feasible inside-money allocations.

Proposition 1 *If $\{y_t, c_t, p_t, h_t\}_{t=0}^{\infty}$ is an incentive-feasible, outside-money allocation, then it is an incentive-feasible, inside-money allocation.*

Proof. As noted above, because type-2 people can hide money, w_t^2 and v_t^2 are weakly increasing. It follows that if (19) and (20) in definition 2 hold, then (15) and (16) in definition 1 hold. And, obviously, (21) implies (17). Thus, it remains to show that (18) holds. We have

$$p_t(2, z, 1, z') = z + z' - c_t(2, z, 1, z') \geq z - c_t(2, z, 1, z'), \qquad (22)$$

where the equality follows from (21), preservation of money holdings in all meetings in outside-money allocations. But then

$$w_t^1[p_t(2, z, 1, z')] \geq w_t^2[z + z' - c_t(2, z, 1, z')] \geq w_t^2[z - c_t(2, z, 1, z')]. \qquad (23)$$

The first inequality follows from the equality in (22) and the first inequality in (20), no defection under the outside-money allocation. The second inequality follows from the fact that w_t^2 is weakly increasing. ■

[8] The specification assumes that the planner can prevent the same type-2 person from getting multiple transfers at the same date. If type-2 people are truly anonymous, then that may be far-fetched. An alternative is to impose that there are no transfers to type-2 people in meetings with the planner. The result below holds for that alternative specification because it would be common to both inside- and outside-money allocations.

The potential advantages of inside money follow from definitions 1 and 2. First, under inside money, in meetings between type-1 consumers and type-2 producers, there is no need to preserve the sum of the states in the meeting. This was the only difference in Cavalcanti and Wallace [3]. Here, if it were the only difference, then it could be overcome under outside money with sufficiently high transfers from the planner to monitored people. However, because a defector under outside money can defect with any outside money held, such transfers are not a perfect substitute for allowing a type-1 person to use a person-specific printing press, a printing press that can be rendered useless if the person defects. In other words, as the model is formulated, planner transfers to type-1 under outside money have to be limited because they increase the payoff to defecting.

The subset result would be completely uninteresting if there were no active-trade, outside-money allocations that satisfy definition 2. That is not the case. Here is an example, adapted from Camera and Corbae [1]. No distinction is made between monitored and non monitored people and the planner does nothing. In every single-coincidence meeting, 1 unit of money trades for the amount of output that keeps the seller indifferent between trading and not trading. (When buyers have no money, there is no trade.) Under such trade, there is a stationary distribution of money holdings that is geometric. In addition, the implied w functions are concave and strictly increasing. The buyer gains from such trade and the seller is indifferent so that participation constraints hold. And the concavity of the value function implies that the seller does not gain from hiding money; obviously, neither does the buyer.

Finally, a special case of the subset result is the (well-known) proposition that perfect monitoring implies no role for money. Perfect monitoring here is $m_1 = 1$. In that case, by definition, an inside-money allocation makes no use of money. And the subset result says that any outside-money allocation can be duplicated by such a non-monetary allocation.

4 Good allocations

Ultimately, we are interested in good allocations. The simplest criterion is a representative-agent criterion, interpreted as expected utility prior to the assignment of types and prior to the assignment of states; namely, $W =$

$\sum_{i=1}^{2} m_i(\pi_0^i v_0^i)$. A little manipulation of (1)-(6) implies that

$$W = \frac{1}{K}\sum_{t=0}^{\infty}\left\{\beta^t \sum_{z}\sum_{z'}\left[\sum_{i=1}^{2}\sum_{j=1}^{2}[m_i m_j \pi_t(z)\pi_t(z')g(y_t(i,z,j,z'))]\right]\right\}, \quad (24)$$

where $g(x) \equiv u(x) - x$. That is, as one would surmise, representative-agent welfare is just the discounted sum of the expected value of the excess of utility of consumption over the disutility of production over all single-coincidence meetings. And, of course, if x^* denotes the maximizer of $g(x)$, then an upper bound on W is $g(x^*)/K(1-\beta)$, the value of W achieved if x^* is produced in every single-coincidence meeting.[9]

Proposition 1 implies that the search for good allocations can be limited to inside-money allocations. Beyond that, I can offer only a few vague conjectures.

According to the criterion W, it is desirable to insulate a person's current prospects from the particular realizations of consumption and production opportunities that the person has experienced. The ability to do that for non monitored people is very limited. As noted above, state transitions in meetings with the planner for such people are limited to non negative transfers which must be weakly increasing in a person's money holdings. Because making such transfers strictly increasing seems to run counter to the goal of insuring the non monitored, a surmise is that such transfers should be lump-sum transfers. However, such transfers tend to be inflationary, and, therefore, have to be limited.[10]

As regards monitored people, it may seem that nothing should depend on their states, something that was actually assumed in [2]. However, that may not be best. If nothing depends on the state of monitored people and if they create money when they are consumers in meetings with non monitored producers, then in order to limit inflation monitored people must produce and acquire (and destroy) money when they are producers in meetings with non monitored people. But, if nothing depends on the state of monitored people, then such production is a gift and is likely to be constrained by participation constraints. Some dependence on states for monitored people could loosen such constraints.

[9]Kocherlakota [8] describes a mechanism that achieves the upper bound in this setting. However, his mechanism is vulnerable to cooperative defection by pairs in meetings.

[10]See Molico [10], Deviatov and Wallace [5], and Deviatov [4] for analyses of transfers in versions of the model with no monitored people.

5 Weakening the recognizability assumption

Given proposition 1, why did we see the evolution of central-bank monopolies on note issue? One possible answer is that the model set out above is in crucial respects off the mark. One questionable feature is the recognizability assumption. Recognizability has for long been on the standard list of desirable properties of money. Like the other properties on this list—for example, divisibility and portability—its appearance suggests that the property is in some sense scarce. Recognizability means the ease with which a genuine object can be distinguished from fakes or counterfeits. Hence, if we are to depart from the recognizability assumption made above, then we have to model counterfeiting and its relationship to inside money and to outside money.

One formulation that permits a weakening of the recognizability assumption is in Williamson and Wright [16]. They model the degree of recognizability of goods in terms of the probability that a person receives a completely informative signal rather than a completely uninformative signal about whether the good is genuine. Nosal and Wallace [11] apply that formulation to outside money.

In many respects, the model in [11] is a special case of the setting described above. There is no meeting with a planner, there is a single outside money, there are no monitored people, and individual money holdings are in the set $\{0, 1\}$. At the beginning of each date, each person can produce a unit of counterfeit money at a cost in terms of utility. In a meeting between a buyer and a seller, first the pair receives a signal: with probability ϕ, the signal is informative and reveals whether the buyer's money is genuine or counterfeit; with probability $1 - \phi$, the signal is completely uninformative. Then the buyer makes a take-it-leave-it lottery offer.

Under those assumptions, Nosal and I show the following: if a positive linear function of the cost of counterfeiting and the probability of getting the informative signal is high enough, then there is a monetary equilibrium in which no counterfeiting occurs; otherwise, there is no monetary equilibrium that satisfies the Cho-Kreps refinement. The Cho-Kreps refinement is applicable because if the uninformative signal is realized, then the only possible equilibrium has pooling. But if there is such pooling, then a holder of genuine money could profitably defect by offering a lower probability of giving up money in exchange for less output, a defection that is not profitable for a holder of counterfeit money because retaining a counterfeit is less valuable than retaining a unit of genuine money.

One main assumption I would make in adapting that model to the setting studied above would be to assume that the probability of receiving the informative signal is lower the greater the number of distinct objects to be recognized. While admittedly a brute force assumption, it goes well with the Williamson-Wright formulation of the recognizability problem. Such an assumption should open the way to overturning the advantages of inside money. Indeed, any way of weakening the recognizability assumption that retains the plausible notion that a system with many distinct monetary objects is more vulnerable to counterfeiting than one with a single uniform object should do that.

If weakening the recognizability assumption eliminates the advantage of inside money, then we would want to describe good outside-money arrangements. As regards the treatment of non monitored people, I have nothing to add to what was said above. In its treatment of monitored people, a good outside-money arrangement would tend to have the planner shift money away from those with a lot of it to those with little of it. That is a way to achieve some separation of a person's current prospects from the realizations they have experienced.[11]

6 Concluding remarks

A weakness of the above presentation is that somewhat general ideas are exposited against the background of a very specific model. Thus, the subset result—incentive-feasible, outside-money allocations are a subset of incentive-feasible, inside-money allocations—would seem to be quite general. For example, I suspect that it survives the introduction of private-information preference or technology shocks of the sort studied in Green [6].[12]

Aside from the subset result, the other ideas that seem not to be special to the particular model are the following. Non monitored people are the

[11]Labeling the h function as the actions of a central bank may seem gratuitous. However, I have not imposed the restriction that the stock of outside money is constant. If it is changing, then that calls for something like central bank activity. Also, the form of a good h function is very dependent on the background environment. Settings with additional private information—about idiosyncratic taste or technology shocks—would tend to make good h functions more closely resemble the operations of a central bank discount window.

[12]A limited analysis of good inside-money allocations in the presence of extreme, private-information, productivity shocks is in [2].

source of a demand for transferable, tangible objects. If monitored people can create such objects in their meetings with non monitored people, objects I label inside money, then their spending is freed from dependence on their recent trades. But, if inside money is to work well, then issuers cannot be allowed to defect and to continue issuing. That, in turn, requires that the inside money of one issuer be distinguishable from inside money issued by others. But because the inside monies get passed around among the non monitored people, such recognizability seems problematic. If sufficiently problematic, then it may be best to have a uniform money and a single issuer and to manage the uniform-money system so as to attain some of the desirable features of inside-money.

Although none of the above ideas is new, putting them together provides a somewhat new perspective on the development of central banking and the 19-th century debates about the regulation of private banking and the management of a central bank monopoly on note issue . Even the background environment seems reasonable for the purpose to which it is put. According to Lloyd Mints (see [9], pages 10, 11), Adam Smith invented the *real bills doctrine* in the following passage:

> When a bank discounts to a merchant a real bill of exchange drawn by a real creditor upon a real debtor, and which, as soon as it becomes due, is really paid by that debtor; it only advances to him a part of the value which he would otherwise be obliged to keep by him unemployed and in ready money for answering occasional demands ([13], page 323).

Smith's real debtor is like a monitored person in the model. Such a person's ability to produce plays the backing role implicit in Smith's description of a *real bill*. And, although the required results are not yet in hand, there is a strong presumption that the model favors the banking school rather than the currency school. The model suggests that there is a role for inside money under the recognizability assumption. However, it gives no support to the notion that inside money can be fruitfully studied using the competitive framework.

The ideas set out above are not only relevant to monetary history. Versions of the inside-outside money issues debated in the 19-th century are still with us. How should central-bank discount windows operate? What role should central banks have in providing intra-day credit? And how should stored value—the modern day equivalent of private banknotes—be regulated? All of these questions are closely related to the 19-th century debates about inside and outside money, debates that have never been settled.

References

[1] G. Camera and D. Corbae, Money and price dispersion. *International Econ. Rev.*, **40** (1999), 965-1008.

[2] R. Cavalcanti and N. Wallace, A model of private banknote issue. *Rev. of Econ. Dynamics,* **2** (January 1999), 104-36.

[3] R. Cavalcanti and N. Wallace, Inside and outside money as alternative media of exchange. *J. of Money, Banking, and Credit,* ***31*** (August, 1999, part 2), 443-57.

[4] A. Deviatov, Money creation in a random-matching model. manuscript, 2003, http://www.nes.ru/˜deviatov.

[5] A. Deviatov and N. Wallace, Another example in which lump-sum money creation is beneficial. *Advances in Macroeconomics*, Vol. 1: No. 1, (2001), Article 1.

[6] E. J. Green, Lending and the smoothing of uninsurable income. In *Contractual Arrangements for Intertemporal Trade* ed by E.C. Prescott and N. Wallace. University of Minnesota Press, Minneapolis 1987, 3-25.

[7] N. Kocherlakota, Money is memory. *J. of Econ. Theory* **81** (August 1998), 232-51.

[8] Kocherlakota, The two-money theorem. *International Econ. Rev.*, **43** (2002), 333-46.

[9] L. Mints, *A History of Banking Theory*, University of Chicago Press, 1945.

[10] M. Molico, The distribution of money and prices in search equilibrium. Unpublished Ph.D. dissertation. The University of Pennsylvania, 1997.

[11] E. Nosal and N. Wallace, A model of counterfeiting, manuscript, 2004.

[12] S. Shi, Money and prices: a model of search and bargaining. *J. of Econ. Theory* **67** (1995), 467-498.

[13] A. Smith, *The Wealth of Nations*, The University of Chicago Press, 1976.

[14] A. Trejos and R. Wright, Search, bargaining, money and prices. *J. of Political Economy* **103** (1995) 118-141.

[15] N. Wallace and T. Zhu, A commodity-money refinement in matching models. *J. of Econ. Theory*, (forthcoming).

[16] S. Williamson and R. Wright, Barter and monetary exchange under private information. *Amer. Econ. Rev.* **84** (1994), 104-123.